Mathematics
Junior High

Grade 9

Teacher Guide

Standards-Based

Papua New Guinea

Department of Education

Mathematics

Junior High

Grade 9

Teacher Guide

Standards-Based

Papua New Guinea

Department of Education

Issued free to schools by the Department of Education

Published in 2020 by the Department of Education, Papua New Guinea

Graphic Design & Layout by David Kuki Gerega

ISBN 978-9980-906-06-9

Contents

Acknowledgements

The Mathematics Teacher Guide for Grade 9 was developed by the Curriculum Development Division of the Department of Education and was coordinated by Kila Tau Gima assisted by Armstrong Rupa and Anda Apex Apule and the Subject Curriculum Group (SCG).

High school teachers, Teacher College lecturers, University Lecturers and other educational experts are acknowledged for their input in the development of this Teacher Guide.

The department also acknowledges Dr Arnold Kukari as a consultant in the realignment and the development of SBC including this Teacher Guide.

Subject Advisory Committee (SAC) and Board of Studies (BOS) are acknowledged for their recommendations and endorsements of this Teacher Guide.

Acronyms

AAL	Assessment As Learning
AFL	Assessment For Learning
AOL	Assessment Of Learning
BoS	Board of Studies
CDD	Curriculum Development Division
CP	Curriculum Panel
CRS	Classroom Response System
DA	Diagnostic Assessment
HOD	Head Of Department
IHD	Integral Human Development
MTDG	Medium Term Development Goals
NGO	Non-Government Organisations
PBA	Performance Based Assessments
PNG	Papua New Guinea
SAC	Subject Advisory Committee
SBC	Standards-Based Curriculum
SBE	Standards-Based Education
SCG	Subject Curriculum Group
SRS	Student Response System
STEAM	Science, Technology, Engineering, Arts and Mathematics
STEM	Science, Technology, Engineering, and Mathematics

Secretary's Message

The aims and goals of SBC is to identify the important knowledge, skills, values and attitudes that all students are expected to acquire and master in order to effectively function in society and actively contribute to its development, student's welfare and enable them to acquire and apply 21st Century skills.

The aims of teaching and learning mathematics is to encourage and enable students to recognize that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

A variety of teaching and learning activities provides students with ideas to motivate students to learn, and make learning relevant, interesting and enjoyable. Teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics.

Learning Mathematics enable students to develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others, develop a critical appreciation of the use of information and communication technology in mathematics appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Teachers are encouraged to integrate Mathematics activities with other subjects, where appropriate, so that students can see the interrelationship between subjects and that the course they are studying provides a holistic education and a pathway for the future.

I commend and approve this Grade 9 Mathematics Teacher Guide to be used in all High Schools throughout Papua New Guinea.

....

UKE W. KOMBRA, PhD
Secretary for Education

Introduction

The aims of teaching and learning mathematics are to encourage and enable students to recognize that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

The curriculum is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways and situations. To support this process, teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics.

Mathematics aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21^{st} Century.

Students should be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

Students' employability will be enhanced through the study and application of STEAM principles. STEAM is an integral component of the core curriculum. All students are expected to study STEAM and use STEAM related skills to solve problems relating to both the natural and the physical environments. The aim of STEAM education is to create a STEAM literate society. It is envisioned that the study of STEAM will motivate students to pursue and take up academic programs and careers in STEAM related fields. STEAM has been embedded in the Mathematics curriculum. Equal opportunities should be provided for all students to learn, apply and master STEAM principles and skills.

Time allocation for Mathematics is **200** minutes for Grade 9.

Structure of the Teacher Guide

There are four main components to this teacher guide. They provide essential information on what all teachers should know and do to effectively implement the Mathematics curriculum.

Part 1 provides generic information to help the teachers to effectively use the teacher guide and the syllabus to plan, teach and assess students' performance and proficiency on the national content standards and grade-level benchmarks. The purpose of the teacher guide, syllabus and teacher guide alignment, and the four pillars of PNG SBC, that is, morals and values education, cognitive and high level thinking, and 21st Century thinking skills, STEAM, and core curriculum are explained to inform as well as guide the teachers so that they align SBE/SBC aims and goals, overarching and SBC principles, content standards, grade-level benchmarks, learning objectives and best practice when planning lessons, teaching, and assessing students.

Part 2 provides information on the strands, units, topics and learning objectives. How topics and learning objectives are derived is explained to the teachers to guide them to use the learning objectives provided for planning, instruction and assessment. And to develop additional topics and learning objectives to meet the learning needs of their students and communities where necessary.

Part 3 provides information on SBC planning to help guide the teachers when planning SBC lessons. Elements and standards for SBC lesson plans are described as well as how to plan for underachievers, use evidence to plan lessons, and use differentiated instruction, amongst other teaching and learning strategies.

Part 4 provides information on standards-based assessment, inclusive of performance assessment and standards, standards-based evaluation, standards-based reporting, and standards-based monitoring. This information should help the teachers to effectively assess, evaluate, report and monitor demonstration of significant aspects of a benchmark.

The above components are linked and closely aligned. They should be connected to ensure that the intended learning outcomes and the expected quality of education standards are achieved. The close alignment of planning, instruction and assessment is critical to the attainment of learning standards.

Purpose of the Teacher Guide

This teacher guide describes what all teachers should know and do to effectively plan, teach, and assess the Grade 9 Mathematics content to enable all students to attain the required learning and proficiency standards. The overarching purpose of this teacher guide is to help teachers to effectively plan, teach, assess, evaluate, report and monitor students' learning and mastery of national and grade-level expectations. That is, the essential knowledge, skills, values and attitudes (KSVAs) described in the content standards and grade-level benchmarks, and their achievement of the national and grade-level proficiency standards.

Ample information with thorough guidelines is provided for the teacher to use to achieve the essential KSVAs embedded in the set national content standards and grade level benchmarks. Thus, the teacher is expected to:

- understand the significance of aligning all the elements of Standards-Based Curriculum (SBC) as the basis for achieving the expected level of education quality;
- effectively align all the components of SBC when planning, teaching, and assessing students' learning and levels of proficiency;
- effectively translate and align the Mathematics syllabi and teacher guide to plan, teach and assess different Mathematics units and topics, and the KSVAs described in the grade-level benchmarks;
- understand the Mathematics national content standards, grade-level benchmarks, and evidence outcomes;
- effectively make sense of the content (KSVAs) described in the Mathematics national content standards and the essential components of the content described in the grade-level benchmarks;
- effectively guide students to progressively learn and demonstrate proficiency on a range of Mathematics skills, processes, concepts, ideas, principles, practices, values and attitudes.
- confidently interpret, translate and use Mathematics content standards and benchmarks to determine the learning objectives and performance standards, and plan appropriately to enable all students to achieve these standards;
- embed the core curriculum in their Mathematics lesson planning, instruction, and assessment to permit all students to learn and master the core KSVAs required of all students;
- provide opportunities for all students to understand how STEAM has and continues to shape the social, political, economic, cultural, and environment contexts and the consequences, and use STEAM principles, skills, processes, ideas and concepts to inquire into and solve problems relating to both the natural and physical (man-made) worlds as well as problems created by STEAM;
- integrate cognitive skills (critical, creative, reasoning, decision-making, and problem-solving skills), high level thinking skills (analysis, synthesis and evaluation skills), values (personal, social, work, health, peace, relationship, sustaining values), and attitudes in lesson planning, instruction and assessment;
- meaningfully connect what students learn in Mathematics with what is learnt in other subjects to add value and enhance students' learning so that they can integrate what they learn and develop in-depth vertical and horizontal understanding of subject content;

- formulate effective SBC lesson plans using learning objectives identified for each of the topics;
- employ SBC assessment approaches to develop performance assessments to assess students' proficiency on a content standard or a component of the content standard described in the grade-level benchmark;
- effectively score and evaluate students' performance in relation to a core set of learning standards or criteria, and make sense of the data to ascertain students' status of progress towards meeting grade-level and nationally expected proficiency standards, and use evidence from the assessment of students' performance to develop effective evidence-based intervention strategies to help students' making inadequate or slow progress towards meeting the grade-level and national expectations to improve their learning and performance.

How to use the Teacher Guide

Teacher Guide provides essential information about what the teacher needs to know and do to effectively plan, teach and assess students learning and proficiency on learning and performance standards. The different components of the teacher guide are closely aligned with SBC principles and practice, and all the other components of PNG SBC. It should be read in conjunction with the syllabus in order to understand what is expected of teachers and students to achieve the envisaged quality of education outcomes.

The first thing teachers should do is to read and understand each of the sections of the teacher guide to help them understand the key SBC concepts and ideas, alignment of PNG SBC components, alignment of the syllabus and teacher guide, setting of content standards and grade-level benchmarks, core curriculum, STEAM, curriculum integration, essential knowledge, skills, values and attitudes, strands, units and topics, learning objectives, SBC lesson planning, and SBC assessment. A thorough understanding of these components will help teachers meet the teacher expectations for implementing the SBC curriculum, and therefore the effective implementation of the Grade 9 Mathematics Curriculum. Based on this understanding, teachers should be able to effectively use the teacher guide to do the following:

Identifying topics from benchmarks'

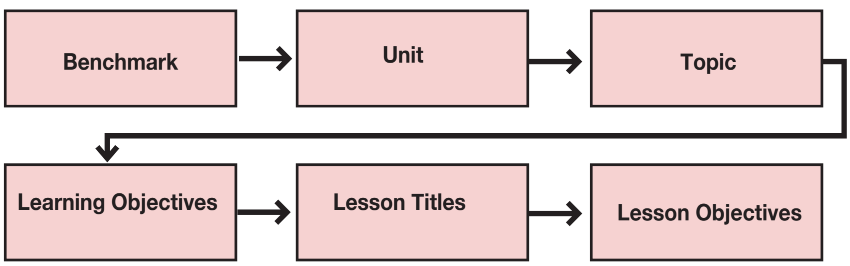
In order to identify the topic from the benchmark, the benchmark needs to be unpack. When unpacking a benchmark, identify what students will be able to know and do in order to mastered the benchmark.

Below is a description of how topics and learning objectives were derived from the grade-level benchmarks.'

1. Write out the benchmark that you want to unpack.
2. Write the verbs (skills/actions) – Higher order thinking skills.
3. Underline or highlight the big idea (content) in the benchmark. The big idea (content) is the topic derived from the benchmark.
4. Write essential questions that would be engaging for students.
5. Develop sub-topics from the big idea (topic).
6. Write learning objectives according to the sub-topics.
7. Write lesson topics from the learning objectives.

Determine Learning Objectives and Lesson Topics

Topics and learning objectives have been identified and described in the Teacher Guide. Lesson objectives are derived from topics that are extracted from the grade-level benchmarks. Lesson topics are deduced from the learning objectives. Teachers should familiarise themselves with this process as it is essential for lesson planning, instruction and assessment. However, depending on the context and students' learning abilities, teachers would be required to determine additional learning objectives and lesson topics. Teachers should use the examples provided in this teacher guide to formulate additional learning objectives and lesson topics to meet the educational or learning needs of their students.



Identify and Teach Grade Appropriate Content

Grade appropriate content has been identified and scoped and sequenced using appropriate content organisation principles. The content is sequenced using the spiraling sequence principles. This sequencing of content will enable students to progressively learn the essential knowledge, skills, values and attitudes as they progress further into their schooling. What students learn in previous grades is reinforced and deepens in scope with an increase in the level of complexity and difficulty in the content and learning activities. It is important to understand how the content is organised so that grade appropriate content and learning activities can be selected, if not already embedded in the benchmarks and learning objectives, to not only help students learn and master the content, but ensure that what is taught is rigorous, challenging, and comparable.

Integrate the Core Curriculum in Lesson Planning, Instruction and Assessment

Teachers should use this teacher guide to help them integrate the core curriculum – values, cognitive and high level skills, 21st Century skills, STEAM principles and skills, and reading, writing, and communication skills in their lesson planning, instruction and assessment. All students in all subjects are required to learn and master these skills progressively through the education system.

Integrate Cognitive, High Level, and 21st Century Skills in Lesson Planning, Instruction and Assessment

Teachers should integrate the cognitive, high level and 21st Century skills in their annual teaching programs, and give prominence to these skills in their lesson preparation, teaching and learning activities, performance assessment, and performance standards for measuring students' proficiency on these skills. Mathematics addresses the skills and processes of solving problems arising in everyday life, society and the workplace. Thus, students will be able to make informed decisions, problem-solving and management knowledge, skills, values and attitudes in Mathematics. This enables them to function effectively in the work and higher education environments as productive and useful citizens of a culturally diverse and democratic society in an interdependent world.

In addition, it envisaged all students attaining expected proficiency levels in these skills and will be ready to pursue careers and higher education academic programs that demand these skills, and use them in their everyday life after they leave school at the end of Grade 12. Teachers should use the teacher guide to help them to effectively embed these skills, particularly in their lesson planning and in the teaching and learning activities as well as in the assessment of students' application of the skills.

Integrate Mathematics Values and Attitudes in Lesson Planning, Instruction and Assessment

In Mathematics, students are expected to learn, promote and use work, relationship, peace, health, social, personal, family, community, national and global values in the work and study environments as well as in their conduct as community, national and global citizens. Teachers should draw from the information and suggestions provided in the syllabus and teacher guide to integrate values and attitudes in their lesson planning, instruction, and assessment. They should report on students' progression towards internalizing different values and attitudes and provide additional support to students who are yet to reach the internalization stage to make positive progress towards this level.

Integrate Science, Technology, Engineering, Arts and Mathematics (STEAM)

Teachers should draw from both the syllabus and teacher guide in order to help them integrate STEAM principles and skills, and methodologies in their lesson planning, instruction and assessment. STEAM teaching and learning happens both inside and outside of the classroom. Effective STEAM teaching and learning requires both the teacher and the student to participate as core investigators and learners, and to work in partnership and collaboration with relevant stakeholders to achieve maximum results. Teachers should use the syllabus, teacher guides and other resources to guide them to plan and implement this and other innovative and creative approaches to STEAM teaching and learning to make STEAM principles and skills learning fun and enjoyable and, at the same time, attain the intended quality of learning outcomes.

Identify and Use Grade and Context Appropriate, Innovative, Differentiated and Creative Teaching and Learning Methodologies

SBC is an eclectic curriculum model. It is an amalgam of strengths of different curriculum types, including behavioural objectives, outcomes, and competency. Its emphasis is on students attaining clearly defined, measurable, observable and attainable learning standards, i.e., the expected level of education quality. Proficiency (competency) standards are expressed as performance standards/criteria and evidence outcomes, that is, what all students are expected to know (content) and do (application of content in real life or related situations) to indicate that they are meeting, have met or exceeded the learning standards. The selection of grade and contextually appropriate teaching and learning methodologies is critical to enabling all students to achieve the expected standard or quality of education. Teaching and learning methodologies must be aligned to the content, learning objective, and performance standard in order for the teacher to effectively teach and guide students towards meeting the performance standard for the lesson. They should be equitable and socially inclusive, differentiate, student-centred, and lifelong. They should enable STEAM principles and skills to be effectively taught and learned by students. Teachers should use the teacher guide to help them make informed decisions when selecting the types of teaching and learning methodologies to use in their teaching of the subject content, including STEAM principles and skills.

Plan Standards-Based Lessons

SBC lesson planning is quite difficult to do. However, this will be easier with more practice and experience over time. Effective SBC lesson plans must meet the required standards or criteria so that the learning objectives and performance standards are closely aligned to attain the expected learning outcomes. Teachers should use the guidelines and standards for SBC lesson planning and examples of SBC lesson plans provided in the teacher guide to plan their lessons. When planning lessons, it is important for teachers to ensure that all SBC lesson planning standards or criteria are met. If standards are not met, instruction will not lead to the attainment of intended performance and proficiency standards. Therefore, students will not attain the national content standards and grade-level benchmarks.

Use Standards-Based Assessment

Standards-Based Assessment has a number of components. These components are intertwined and serve to measure evaluate, report, and monitor students' achievement of the national and grade-level expectations, i.e., the essential knowledge, skills, values and attitudes they are expected to master and demonstrate proficiency on. Teachers should use the information and examples on standards-based assessment to plan, assess, record, evaluate, report and monitor students' performance in relation to the learning standards.

Make informed Judgments About Students' Learning and Progress Towards Meeting Learning Standards

Teachers should use the teacher guide to effectively evaluate students' performance and use the evidence to help students to continuously improve their learning as well as their classroom practice.

It is important that teachers evaluate the performance of students in relation to the performance standards and progressively the grade-level benchmarks and content standards to make informed judgments and decisions about the quality of their work and their progress towards meeting the content standards or components of the standards. Evaluation should not focus on only one aspect of students' performance. It should aim to provide a complete picture of each student's performance. The context, inputs, processes, including teaching and learning processes, and the outcomes should be evaluated to make an informed judgment about each student's performance, Teachers should identify the causal factors for poor performance, gaps in students learning, gaps in teaching, teaching and learning resource constraints, and general attitude towards learning. Evidence-based decisions can then be made regarding the interventions for closing the gaps to allow students to make the required progress towards meeting grade-level and national expectations.

Prepare Students' Performance Reports

Reporting of students' performance and progress towards the attainment of learning standards is an essential part of SBC assessment. Results of students' performance should be communicated to particularly the students and their parents to keep them informed of students' academic achievements and learning challenges as well as what needs to be done to enable the students' make positive progress towards meeting the proficiency standards and achieve the desired level of education quality. Teachers should use the information on the reporting of students' assessment results and the templates provided to report the results of students' learning.

Monitor Students' Progress Towards Meeting the National Content Standards and Grade-Level Benchmarks

Monitoring of student's progress towards the attainment of learning standards is an essential component of standards-based assessment. It is an evidence-based process that involves the use of data from students' performance assessments to make informed judgements about students' learning and proficiency on the learning standards or their components, identify gaps in students' learning and the causal factors, set clear learning improvement targets, and develop effective evidence-based strategies (including pre-planning and re-teaching of topics), set clear time-frames, and identify measures for measuring students' progress towards achieving the learning targets.

Teachers should use the teacher guide to help them use data from students' performance assessments to identify individual students' learning weaknesses and develop interventions, in collaboration with each student and his/her parents or guardians, to address the weaknesses and monitor their progress towards meeting the agreed learning goals.

Develop additional Benchmarks

Teachers can develop additional benchmarks using the examples in the teacher guide to meet the learning needs of their students and local communities. However, these benchmarks will not be nationally assessed as these are not comparable. They are not allowed to set their own content standards or manipulate the existing ones. The setting of national content standards is done at the national level to ensure that required learning standards are maintained and monitored to sustain the required level of education quality.

Avoid Standardisation

The implementation of Grade 9 Mathematics curriculum must not be standardised. SBC does not mean that the content, lesson objectives, teaching and learning strategies, and assessment are standardised. This is a misconception and any attempt to standardise the components of curriculum without due consideration of the teaching and learning contexts, children's backgrounds and experiences, and different abilities and learning styles of children will be counterproductive. It will hinder students from achieving the expected proficiency standards and hence, high academic standards and the desired level of education quality. That is, they should not be applied across all contexts and with all students, without considering the educational needs and the characteristics of each context. Teachers must use innovative, creative, culturally relevant, and differentiated teaching and learning approaches to teach the curriculum and enable their students to achieve the national content standards and grade-level benchmarks. And enable all students to experience success in learning the curriculum and achieve high academic standards.

What is provided in the syllabus and teacher guide are not fixed and can be changed. Teachers should use the information and examples provided in the syllabus and the teacher guide to guide them to develop, select, and use grade, context, and learner appropriate content, learning objectives, teaching and learning strategies, and performance assessment and standards. SBC is evidence-based hence decisions about the content, learning outcomes, teaching and learning strategies, students' performance, and learning interventions should be based on evidence. Teaching and learning should be continuously improved and effectively targeted using evidence from students' assessment and other sources.

Syllabus and Teacher Guide Alignment

A teacher guide is a framework that describes how to translate the content standards and benchmarks (learning standards) outlined in the syllabus into units and topics, learning objectives, lesson plans, teaching and learning strategies, performance assessment, and measures for measuring students' performance (performance standards). It expands the content overview and describes how this content identified in the content standards and their components (essential KSVAs) can be translated into meaningful and evidence-based teaching topics and learning objectives for lesson planning, instruction and assessment. It also describes and provides examples of how to evaluate and report on students' attainment of the learning standards, and use evidence from the assessment of students' performance to develop evidence-based interventions to assist students who are making slow progress towards meeting the expected proficiency levels to improve their performance.

Grade 9 Mathematics comprises of the Syllabus and Teacher Guide. These two documents are closely aligned, complimentary and mutually beneficial. They are the essential focal points for teaching and learning the essential Mathematics knowledge, skills, values and attitudes.

Syllabus and Teacher Guide Alignment	
Syllabus	**Teacher Guide**
Outlines the ultimate aim and goals, and what to teach and why teach it	Describes how to plan, teach, and assess students' performance
• Overarching and SBC principles • Content overview • Core curriculum • Essential knowledge, skills, values and attitudes • Strands and units • Evidence outcomes • Content standards and grade-level • benchmark • Overview of assessment, evaluation, and Reporting	• Determine topics for lesson planning, instruction and assessment • Formulate learning objectives • Plan SBC lesson plans • Select teaching and learning strategies • Implement SBC assessment and evaluation • Implement SBC reporting and monitoring

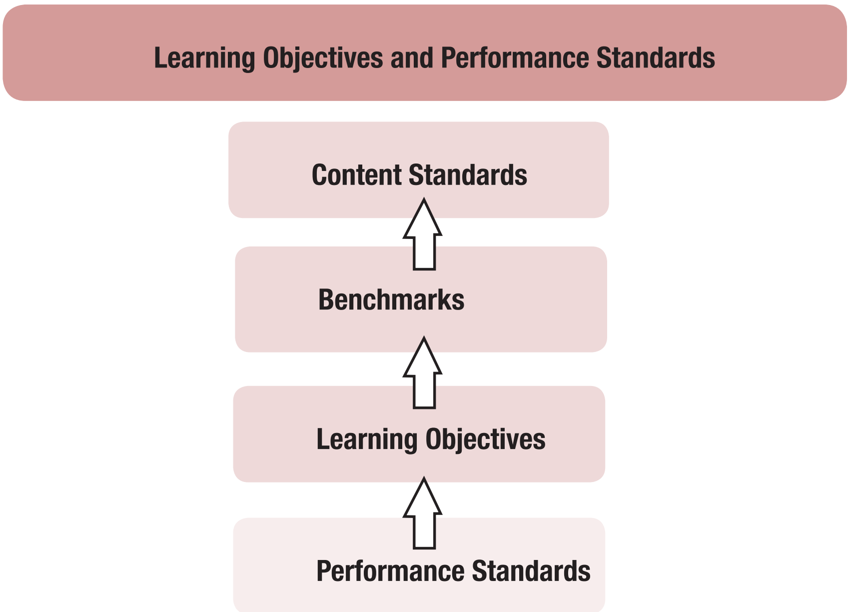
The syllabus outlines the ultimate aim and goals of SBE and SBC, what is to be taught and why it should be learned by students, the underlying principles and articulates the learning and proficiency standards that all students are expected to attain. On the other hand, the teacher guide expands on what is outlined in the syllabus by describing the approaches or the how of planning, teaching, learning, and assessing the content so that the intended learning outcomes are achieved.

This teacher guide should be used in conjunction with the syllabus. Teachers should use these documents when planning, teaching and assessing Grade 9 Mathematics content.

Teachers will extract information from the syllabus (e.g., content standards and grade-level benchmarks) for lesson planning, instruction and is for measuring students' attainment a content standard as well as progress to the next grade of schooling.

Learning and Performance Standards Alignment

Content Standards, Benchmarks, Learning Objectives, and Performance Standards are very closely linked and aligned (see below). There is a close linear relationship between these standards. Students' performance on a significant aspect of a benchmark (KSVA) is measured against a set of performance standards or criteria to determine their level of proficiency using performance assessment. Using the evidence from the performance assessment, individual student's proficiency on the aspect of the benchmark assessed and progression towards meeting the benchmark and hence the content standard are then determined.



Effective alignment of these learning standards and all the other components of PNG SBE and SBC (ultimate aim and goals, overarching, SBC and subject-based principles, core curriculum, STEAM, and cognitive, high level, and 21st Century skills) is not only critical but is also key to the achievement of high academic standards by all students and the intended level of education quality. It is essential that teachers know and can do standards alignment when planning, teaching, and assessing students' performance so that they can effectively guide their students towards meeting the grade-level benchmarks (grade expectations) and subsequently the content standards (national expectations).

Learning and Performance Standards

Standards-Based Education (SBE) and SBE are underpinned by the notion of quality. Standards define the expected level of education quality that all students should achieve at a particular point in their schooling. Students' progression and achievement of education standard (s) are measured using performance standards or criteria to determine their demonstration or performance on significant aspects of the standards and therefore their levels of proficiency or competency. When they are judged to have attained proficiency on a content standard or benchmark or components of these standards, they are then deemed to have met the standard(s) that is, achieved the intend level of education quality.

Content standards, benchmarks, and learning objectives are called learning standards while performance and proficiency standards (evidence outcomes) can be categorised as performance standards. These standards are used to measure students' performance, proficiency, progression and achievement of the desired level of education quality. Teachers are expected to understand and use these standards for lesson planning, instruction and assessment

Content Standards

Content standards are evidence-based, rigorous and comparable regionally and globally. They have been formulated to target critical social, economic, political, cultural, environment, and employable skills gaps identified from a situational analysis. They were developed using examples and experiences from other countries and best practice, and contextualized to PNG contexts.

Content standards describe what (content - knowledge, skills, values, and attitudes) all students are expected to know and do (how well students must learn and apply what is set out in the content standards) at each grade-level before proceeding to the next grade. These standards are set at the national level and thus cannot be edited or changed by anyone except the National Subject-Based Standards Councils. Content Standards:

- are evidenced-based,
- are rigorous and comparable to regional and global standards,
- are set at the national level,
- state or describe the expected levels of quality or achievement,
- are clear, measurable and attainable,
- are linked to and aligned with the ultimate aim and goals of SBE and SBC and overarching and SBC principles,
- delineate what matters, provide clear expectations of what students should progressively learn and achieve in school, and guide lesson planning, instruction, assessment,
- comprise knowledge, skills, values, and attitudes that are the basis for quality education,
- provide teachers a clear basis for planning, teaching, and assessing lessons,
- provide provinces, districts, and schools with a clear focus on how to develop and organise their instruction and assessment programs as well as the content that they will include in their curriculum.

Benchmarks

Benchmarks are derived from the content standards and benchmarked at the grade-level. Benchmarks are specific statements of what students should know (i.e., essential knowledge, skills, values or attitudes) at a specific grade-level or school level. They provide the basis for measuring students' attainment of a content standard as well as progress to the next grade of schooling.

Grade-level benchmarks:

- are evidenced-based,
- are rigorous and comparable to regional and global standards,
- are set at the grade level,
- are linked to the national content standards,
- are clear, measurable, observable and attainable,
- articulate grade level expectations of what students are able to demonstrate to indicate that they are making progress towards attaining the national content standards,
- provide teachers a clear basis for planning, teaching, and assessing lessons,
- state clearly what students should do with what they have learned at the end of each school-level,
- enable students' progress towards the attainment of national content standards to be measured, and
- enable PNG students' performance to be compared with the performance of PNG students with students in other countries.

Approach for Setting National Content Standards and Grade-Level Benchmarks

Strand ⇨ **Content Standards** ⇨ **Grade- Level Benchmarks**

Development of Additional Benchmarks

Teachers should develop additional benchmarks to meet the learning needs of their students. They should engage their students to learn about local, provincial, national and global issues that have not been catered for in the grade-level benchmarks but are important and can enhance students' understanding and application of the content. However, it is important to note that these benchmarks will not be nationally examined as they are not comparable. Only the benchmarks developed at the national level will be tested. This does not mean that teachers should not develop additional benchmarks. An innovative, reflect, creative and reflexive teacher will continuously reflect on his/her classroom practice and use evidence to provide challenging, relevant, and enjoyable learning opportunities for his/her students to build on the national expectations for students.

Teachers should follow the following process when developing additional grade-level benchmarks

Benchmark Development Process

Determine what needs to be taught (essential knowledge, skills, values and attitudes) → Determine what needs to be taught (essential knowledge, skills, values and attitudes) → Determine what needs to be taught (essential knowledge, skills, values and attitudes)

Learning Objectives

Learning or instructional Objectives are precise statements of educational intent. They are formulated using a significant aspect or a topic derived from the benchmark, and is aligned with the educational goals, content standards, benchmarks, and performance standards. Learning objectives are stated in outcomes language that describes the products or behaviours that will be provided by students. They are stated in terms of measurable and observable student behaviour. For example, students will be able to identify all the main towns of PNG using a map.

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the **"be able to do"** of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They are explicit definitions of what students **must do to demonstrate proficiency or competency at a specific level on the content standards.** Performance standards:

- measure students' performance and proficiency (**using performance indicators**) in the use of a specific knowledge, skill, value, or attitude in real life or related situations.
- provide the basis (**performance indicators**) for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude.
- are used to plan for individual instruction to help students not yet meeting expectations (**desired level of mastery and proficiency**) to make adequate progress towards the full attainment of benchmarks and content standards
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards.

Proficiency Standards

Proficiency standards describe what all students in a particular grade or school level can do at the end of a strand, or unit. These standards are sometimes called evidence outcomes because they indicate if students can actually apply or use what they have learnt in real life or similar situations. They are also categorized as benchmarks because that is what all students are expected to do before exiting a grade or are deemed ready for the next grade.

Core Curriculum

A core set of common learnings (knowledge, skills, values, and attitudes) are integrated into the content standards and grade-level benchmarks for all subjects. This is to equip all students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in modern/postmodern work places, higher-education programs and to be productive, responsible, considerate, and harmonious citizens. Common set of learning are spirally sequenced from Preparatory to Grade 12 to deepen the scope and increase the level of difficulty in the learning activities so that what is learned is reinforced at different grade levels.

The core curriculum includes;

- cognitive (thinking) skills (refer to the syllabus for a list of these skills),
- reasoning, decision-making and problem-solving skills,
- high level thinking skills (analysis, synthesis and evaluation skills),
- 21st Century skills (refer to illustrative list in the appendix 2),
- reading, writing and communication skills,
- STEAM principles and skills,
- essential values and attitudes (core personal and social values, and sustaining values), and
- spiritual values and virtues.

The essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship.

All teachers are expected to include the core learnings in their lesson planning, teaching, and assessment of students in all their lessons. They are expected to foster, promote and model the essential values and attitudes as well as the spiritual values and virtues in their conduct, practice, appearance, and their relationships and in their professional and personal lives. In addition, teachers are expected to mentor, mould and shape each student to evolve and possess the qualities envisioned by society.

Core values and attitudes must not be taught in the classroom only; they must also be demonstrated by students in real life or related situations inside and outside of the classroom, at home, and in everyday life. Likewise, they must be promoted, fostered and modelled by the school community and its stakeholders, especially parents. A holistic of school approach to values and attitudes in teaching, promoting and modelling is critical to students and the whole school community to internalise the core values and attitudes and make them habitual in their work and school place, and in everyday life. Be it work values, relationship values, peace values, health values, personal and social values, or religious values, teachers should give equal prominence to all common learnings in their lesson planning, teaching, assessment, and learning interventions. Common learnings must be at the heart of all teaching and extra-curricular programs and activities.

Science, Technology, Engineering, Arts and Mathematics

STEAM education is an integrated, multidisciplinary approach to learning that uses science,technology, engineering, arts and mathematics as the basis for inquiring about how STEAM has and continues to change and impact the social, political, economic, cultural and environmental contexts and identifying and solving authentic (real life) natural and physical environment problems by integrating STEAM-based principles, cognitive, high level and 21st Century skills and processes, and values and attitudes.

Mathematics is focused on both goals of STEAM rather than just the goal of problem-solving. This is to ensure that all students are provided opportunities to learn, integrate, and demonstrate proficiency on all essential STEAM principles, processes, skills, values and attitudes to prepare them for careers, higher education and citizenship.

Through STEAM education students will be able to:

(i) examine and use evidence to draw conclusions about how STEAM has and continues to change the social, political, economic, cultural and environmental contexts.
(ii) Investigate and draw conclusions on the impact of STEAM solutions to problems on the social, political, economic, cultural and environmental contexts.
(iii) Identify and solve problems using STEAM principles, skills, concepts, ideas and process.
(iv) Identify, analyse and select the best solution to address a problem.
(v) build prototypes or models of solutions to problems.
(vi) replicate a problem solution by building models and explaining how the problem was or could be solved.
(vii) test and reflect on the best solution chosen to solve a problem.
(viii) collaborate with others on a problem and provide a report on the process of problem solving used to solve the problem.
(ix) use skills and processes learnt from lessons to work on and complete STEAM projects.
(x) demonstrate STEAM principles, skills, processes, concepts and ideas through simulation and modelling.
(xi) explain the significance of values and attitudes in problem-solving.

STEAM is a multidisciplinary and integrated approach to understanding how science,technology, engineering, arts and mathematics shape and are shaped by our material, intellectual, cultural, economic, social, political and environmental contexts. And for teaching students the essential in demand cognitive, high level and 21st century skills, values and attitudes, and empower them to effectively use these skills and predispositions to identify and solve problems relating to the natural and physical environments as well as the impact of STEAM-based solutions on human existence and livelihoods, and on the social, political, economic, cultural, and environmental systems.

STEAM disciplines have and continue to shape the way we perceive knowledge and reality, think and act, our values, attitudes, and behaviours, and the way we relate to each other and the environment. Most of the things we enjoy and consume are developed using STEAM principles, skills, process, concepts and ideas.

Things humans used and enjoyed in the past and at present are developed by scientists, technologists, engineers, artists and mathematicians to address particular human needs and wants. Overtime, more needs were identified and more products were developed to meet the ever changing and evolving human needs. What is produced and used is continuously reflected upon, evaluated, redesigned, and improved to make it more advanced, multi-purpose, fit for purpose, and targeted towards not only improving the prevailing social, political,economic, cultural and environmental conditions but also to effectively respond to the evolving and changing dynamics of human needs and wants. And, at the same time, solutions to human problems and needs are being investigated and designed to address problems that are yet to be addressed and concurred. This is an evolving and ongoing problem-solving process that integrates cognitive, high level, and 21st Century skills, and appropriate values and attitudes.

STEAM is a significant framework and focal point for teaching and guiding students to learn, master and use a broad range of skills and processes required to meet the skills demands of PNG and the 21st Century. The skills that students will learn will reflect the demands that will be placed upon them in a complex, competitive, knowledge-based, information-age, technology- driven economy and society. These skills include cognitive (critical, synthetic, creative, reasoning, decision-making, and problem-solving) skills, high level (analysis, synthesis and evaluation) skills and 21st Century skills (see Appendix 4). Knowledge-based, information, and technology driven economies require knowledge workers not technicians. Knowledge workers are lifelong learners, are problem solvers, innovators, creators, critical and creative thinkers, reflective practitioners, researchers (knowledge producers rather than knowledge consumers), solutions seekers, outcomes oriented, evidence-based decision makers, and enablers of improved and better outcomes for all.

STEAM focuses on the skills and processes of problem solving. These skills and processes are at the heart of the STEAM movement and approach to not only problem solving and providing evidence-based solutions but also the development and use of other essential cognitive, high level and 21st Century skills. These skills are intertwined and used simultaneously to gain a broader understanding of the problems to enable creative, innovative, contextually relevant, and best solutions to be developed and implemented to solve the problems and attain the desired outcomes. It is assumed that by teaching students STEAM-based problem-solving skills and providing learning opportunities inside and outside the classroom will motivate more of them to pursue careers and academic programs in STEAM related fields thus, closing the skills gaps and providing a pool of cadre of workers required by technology, engineering, science, and mathematics-oriented industries.

Although, STEAM focuses on the development and application of skills in authentic (real life) contexts, for example the use of problem- solving skills to identify and solve problems relating to the natural and physical worlds, it does not take into account the significant influence values and attitudes have on the entire process of problem solving. Values and attitudes are intertwined with knowledge and skills. Knowledge, skills, values and attitudes are inseparable. Decisions about skills and processes of skills development and application are influenced by values and attitudes (mindset) that people hold. In the same light, the use of STEAM principles, processes and skills to solve problems in order to achieve the outcomes envisaged by society are influenced by values and the mindset of those who have identified and investigated the problem as well as those who are affected by the problem and will benefit from the outcome.

STEAM Problem-Solving Methods and Approaches

Problem-solving involves the use of problem-solving methods and processes to identify and define a problem, gather information to understand its causes, draw conclusions, and use the evidence to design and implement solutions to address it. Even though there are many different problem-solving methods and approaches, they share some of the steps of problem-solving, such as;

- identifying the problem,
- understanding the problem by collecting data,
- analyse and interpret the data,
- draw conclusions,
- use data to consider possible solutions,
- select the best solution,
- test the effectiveness of the solution by trialling and evaluating it, and
- review and improve the solution.

STEAM problem solving processes go from simple and technical to advance and knowledge-based processes. However, regardless of the type of process used, students should be provided opportunities to learn the essential principles and processes of problem solving and, more significantly, to design and create a product that addressed a real problem and meets a human need.

The following are some of the STEAM problem solving processes.

1. Engineering and Technology Problem Solving Methods and Approaches

Engineering and technology problem-solving methods are used to identify and solve problems relating to the physical world using the design process. The following are some of the methods and approaches used to solve engineering and technology related problems.

Parts Substitution
It is the most basic of the problem-solving methods. It simply requires the parts to be substituted until the problem is solved.

Diagnostics
After identifying a problem, the technician would run tests to pinpoint the fault. The test results would be used either as a guide for further testing or for replacement of a part, which also need to be tested. This process continues until the solution is found and the device is operating properly.

Troubleshooting
Troubleshooting is a form of problem solving, often applied to repair failed products or processes.

Reverse Engineering
Reverse engineering is the process of discovering the technological principles underlying the design of a device by taking the device apart, or carefully tracing its workings or its circuitry. It is useful when students are attempting to build something for which they have no formal drawings or schematics.

Divide and Conquer
Divide and conquer is the technique of breaking down a problem into sub-problems, then breaking the sub-problems down even further until each of them is simple enough to be solved. Divide and conquer may be applied to all groups of students to tackle sub-problems of a larger problem, or when a problem is so large that its solution cannot be visualised without breaking it down into smaller components.

Extreme Cases
Considering "extreme cases"-envisioning the problem in a greatly exaggerated or greatly simplified form, or testing using extreme condition – can often help to pinpoint a problem. An example of the extreme-case method is purposely inputting an extremely high number to test a computer program.

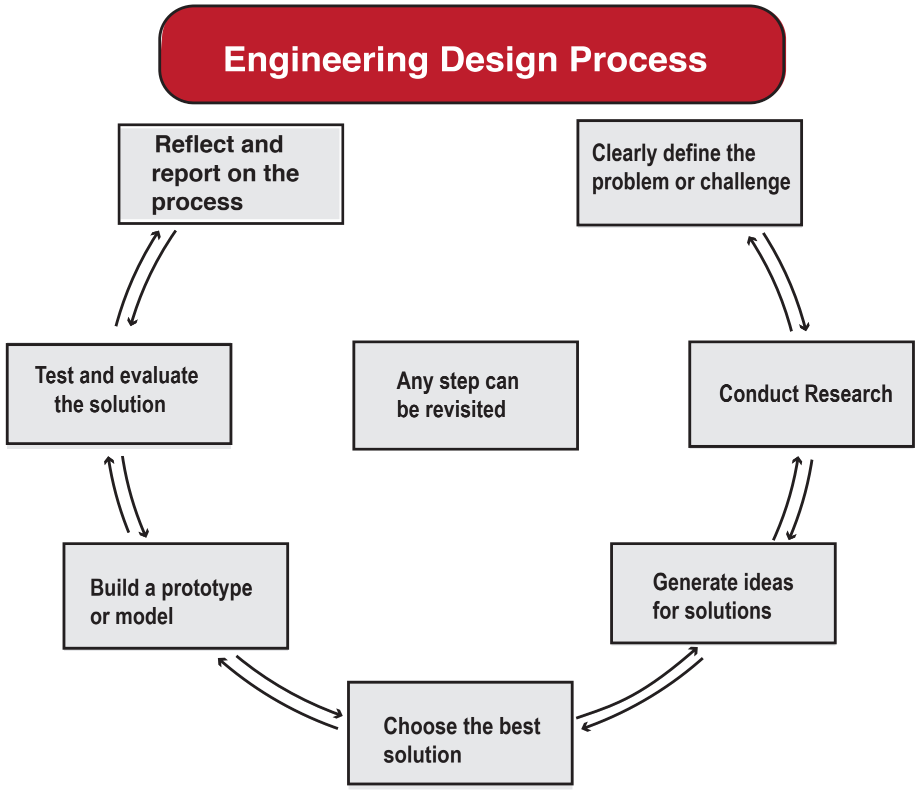
Trial and Error
The trial and error method involve trying different approaches until a solution is found. It is often used as a last resort when other methods have been exhausted.

- Test and evaluate the solution.
- Repeat steps as necessary to modify the design or correct faults.
- Reflect and report on the process.

2. Engineering Design Process

Technological fields use the engineering design process to identify and define the problem or challenge, investigate the problem, collect and analyse data, and use the data to formulate potential solutions to the problem, analyse each of the solutions in terms of its strengths and weaknesses, and choose the best solution to solve the problem. It is an open-ended problem-solving process that involves the full planning and development of products or services to meet identified needs. It involves a sequence of steps as illustrated.

1) Analyse the context and background, and clearly define the problem.
2) Conduct research to determine design criteria, financial or other constraints, and availability of materials.
3) Generate ideas for potential solutions, using processes such as brainstorming and sketching.
4) Choose the best solution.
5) Build a prototype or model.
6) Test and evaluate the solution.
7) Repeat steps as necessary to modify the design or correct faults.
8) Reflect and report on the process.



STEAM-Based Lesson planning

Effective STEAM lesson planning is key to the achievement of expected STEAM outcomes. STEAM skills can be planed and taught using separate STEAM-based lesson plans or integrated into the standards-based lesson plans. To effectively do this, teachers should know how to write effective standards and STEAM-based lesson plans.

An example of a STEAM-based lesson plan is provided in appendix.
Teachers should use this to guide them to integrate STEAM content and teaching, learning and assessment strategies into their standards-based lesson plans.

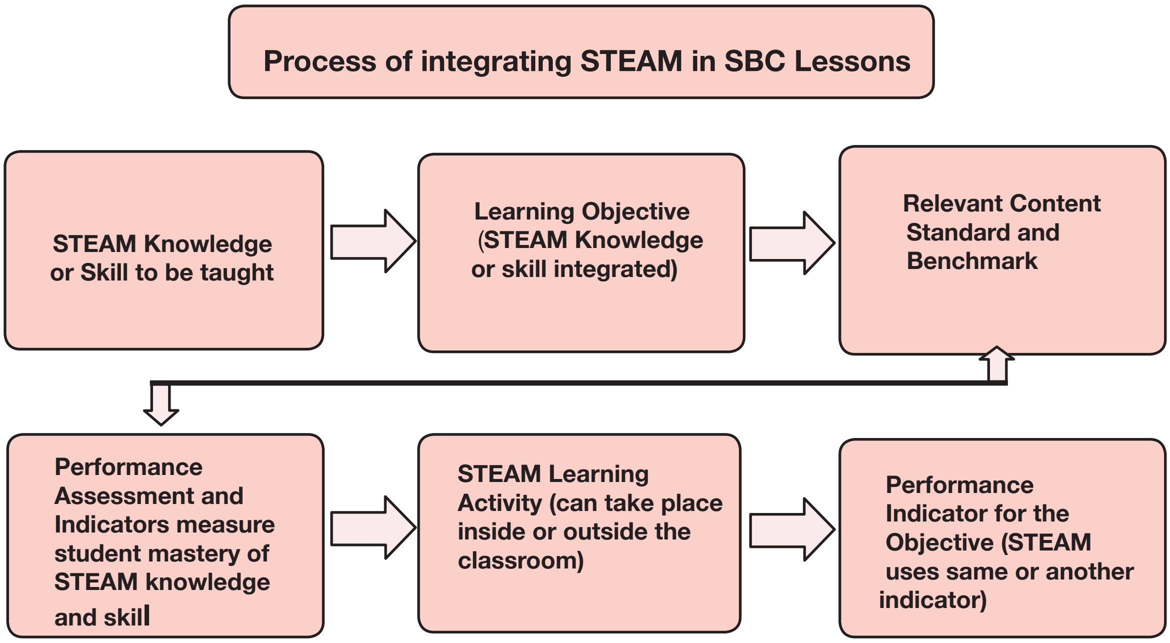
Integration of STEAM problem-solving skills into standards-based lesson plans

Knowing how to integrate STEAM problem-solving skills, principles, values and attitudes as well as STEAM teaching, learning, and assessment strategies into standards-based lesson plans is essential for achieving the desired STEAM learning outcomes. When integrating STEAM problem-solving skills into the standards-based lesson plans, teachers should ensure that these skills are not only effectively aligned to the learning objective and performance standards, they must also be effectively taught and assessed.

STEAM principles and problem-solving skills are integrated into the content standards and grade-level benchmarks. A list of these skills, including 21st Century skills, is provided in the syllabus. Teachers should ensure that these skills are integrated in their standards-based lesson plans, taught and assessed to determine students' level of proficiency on each skill or specific components of the skill.

Teachers are expected to integrate the essential STEAM principles, processes, skills, values and attitudes described in the grade 9 benchmarks when formulating their standards-based lesson plans. Opportunities should be provided inside and outside of the classroom for students to learn, explore, model and apply what they learn in real life or related situations. These learning experiences will enable students to develop a deeper understanding of STEAM principles, processes, skills, values and attitudes and appreciate their application in real life to solve problems.

Teachers should use the following process as guide to integrate STEAM principles and problem-solving skills into the standards-based lesson plans.



Steps for integrating STEAM problem-solving principles and skills into Standards-Based lesson plans.

Step 1: Identify the STEAM knowledge or skill to be taught (from the table of KSVAs for each content standard and benchmark). This is captured in the learning objective stated in the standards-based lesson plan.

Step 2: Develop and include a performance standard or indicator for measuring student mastery of the STEAM knowledge or skill (e.g. level of acceptable competency or proficiency) if this is different from the one already stated in the lesson plan.

Step 3: Develop student learning activity (An activity that will provide students the opportunity to apply the STEAM knowledge or skill specified by the learning objective and appropriate statement of the standards). Activity can take place inside or outside of the classroom, and during or after school hours.

Step 4: Develop and use performance descriptors (standards or indicators) to analyse students' STEAM related behaviours and products (results or outcomes), which provide evidence that the student has acquired and mastered the knowledge or skill of the learning objective specified by the indicator (s) of the standard(s).

STEAM Teaching Strategies

STEAM education takes place in both formal and informal classroom settings. It takes place during and after school hours. It is a continuous process of inquiry, data analysis, making decisions about interventions, and implementing and monitoring interventions for improvements.

There are a variety of STEAM teaching strategies. However, teaching strategies selected must enable teachers to guide students to use the engineering and artistic design processes to identify and solve natural and physical environment problems by designing prototypes and testing and refining them to effectively mitigate the problems identified. The following are some of the strategies that could be used to utilise the STEAM approach to solve problems and coming up with technological solutions.

1. Inquiry-Based Learning
2. Problem-Based Learning
3. Project-based learning
4. Collaborative Learning

Collaborative learning involves individuals from different STEAM disciplines and expertise in a variety of STEAM problem solving approaches working together and sharing their expertise and experiences to inquire into and solve a problem.

Teachers should plan to provide students opportunities to work in collaboration and partnership with experts and practitioners engaged in STEAM related careers or disciplines to learn first-hand about how STEAM related skills, processes, concepts, and ideas are applied in real life to solve problems created by natural and physical environments. Collaborative learning experiences can be provided after school or during school holidays to enable students to work with STEAM experts and practitioners to inquiry and solve problems by developing creative, innovative and sustainable solutions. Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path. Developing STEAM partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations can enhance students' learning and application of STEAM problem solving principles and skills.

Some examples of STEAM-related partnership experiences may include:

- Participatory Learning
- Group-Based Learning
- Task Oriented Learning
- Action Learning
- Experiential Learning
- Modelling
- Simulation

STEAM Learning Strategies

Teachers should include in their lesson plans STEAM learning activities. These activities should be aligned to principle or a skill planned for students to learn and demonstrate proficiency at the end of the lesson to expose students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.

Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.

STEAM-Based Assessment

STEAM-based assessment is closely linked to standards-based assessment where assessment is used to assess students' level of competency or proficiency of a specific knowledge, skill, value, or attitude taught using a set of performance standards (indicators or descriptors). The link also includes the main components such as the purpose, the assessment principles and assessment strategies and tools. In STEAM-based assessment, assessments are designed for what students should know and be able to do. In STEAM learning, students are assessed in a variety of ways including portfolios, project/problem-based assessments, backwards design, authentic assessments, or other student-centered approaches.

When planning and designing the assessment, teachers should consider the authenticity of the assessment by designing an assessment that relates to a real world task or discipline specific attributes such as simulation, role play, placement assessment, live projects and debates. These tasks should make the activity meaningful to the student, and therefore be motivating as well as developing employability skills and discipline specific attributes.

Effective STEAM-Based Assessment Strategies

The following are the six assessment tools and strategies to impact teaching and learning as well as help teachers foster 21st Century learning environment in their classrooms.

1. Rubrics
2. Performance-Based Assessments (PBAs)
3. Portfolios
4. Student self-assessment
5. Peer-assessment
6. Student Response Systems (SRS).

Although the list does not include all innovative assessment strategies, it includes what we think are the most common strategies, and ones that may be particularly relevant to the educational context of developing countries in this 21st Century. Many of the assessment strategies currently in use fit under one or more of the categories discussed. Furthermore, it is important to note that these strategies also connect in a variety of ways.

1. Rubrics

Rubrics are both a tool to measure students' knowledge and ability as well as an assessment strategy. A rubric allows teachers to measure certain skills and abilities not measurable by standardized testing systems that assess discrete knowledge at a fixed moment in time. Rubrics are also frequently used as part of other assessment strategies including; portfolios, performances, projects, peer-review and self-assessment which are also elaborated in this section.

2. Performance-Based Assessments

Performance-Based Assessments (PBA), also known as project-based or authentic assessments, are generally used as a summative evaluation strategy to capture not only what students know about a topic, but if they have the skills to apply that knowledge in a "real-world" situation.

By asking them to create an end product. PBA pushes students to synthesize their knowledge and apply their skills to a potentially unfamiliar set of circumstances that is likely to occur beyond the confines of a controlled classroom setting.

The implementation of performance-based assessment strategies can also impact other instructional strategies in the classroom.

3. Portfolio Assessment

Portfolios are a collection of student work gathered over time that is primarily used as a summative evaluation method. The most salient characteristic of the portfolio assessment is that rather than being a snapshot of a student's knowledge at one point in time (like a single standardized test), it highlights student effort, development, and achievement over a period of time; portfolios measure a student's ability to apply knowledge rather than simply regurgitate. They are considered both student-centred and authentic assessments of learning.

4. Self-assessment

While the previous assessment tools and strategies listed in this report generally function as summative approaches, self-assessment is generally viewed as a formative strategy, rather than one used to determine a student's final grade. Its main purpose is for students to identify their own strengths and weakness and to work to make improvements to meet specific criteria.
Self-assessment occurs when students judge their own work to improve performance as they identify discrepancies between current and desired performance. In this way, self-assessment aligns well with standards- based education because it provides clear targets and specific criteria against which students or teachers can measure learning.

Self-assessment is used to promote self-regulation, to help students reflect on their progress and to inform revisions and improvements on a project or paper. In order for self-assessment to be truly effective four conditions must be in place: the self-assessment criteria is negotiated between teachers and students, students are taught how to apply the criteria, students receive feedback on their self-assessments and teachers help students use assessment data to develop an action plan.

5. Peer assessment

Peer assessment, much like self-assessment, is a formative assessment strategy that gives students a key role in evaluating learning. Peer assessment approaches can vary greatly but, essentially process develops both the assessor and assessee's skills and knowledge.

The primary goal for using peer assessment is to provide feedback to learners. This strategy may be particularly relevant in classrooms with many students per teacher since student time will always be more plentiful than teacher time. Although any single student's feedback may not be rich or in-depth as tea her's feedback, the research suggests that peer assessment can improve learning.

6. Student Response System

Student response system (SRS), also known as classroom response (CRS), audience response system (ARS) is a general term that refers to a variety of technology-based formative assessment tools that can be used to gather student-level data instantly in the classroom. Through the combination of hardware, (voice recorders, PC, internet connection, projector and screen) and software.

Teachers can ask students a wide range of questions (both closed and open ended), where students can respond quickly and anonymously, and the teacher can display the data immediately and graphically. The use of technology also includes a use of video which examines how a range of strategies can be used to assess students' understanding.

The value of SRS comes from teachers analysing information quickly and then devising real-time instructional solutions to maximize student learning. This includes a suggested approach to help teachers and trainers assess learning.

Curriculum Integration

What is Curriculum Integration?

Curriculum integration is making connections in learning across the curriculum. The ultimate aim of curriculum integration is to act as a bridge to increase students' achievement and engage in relevant curriculum.

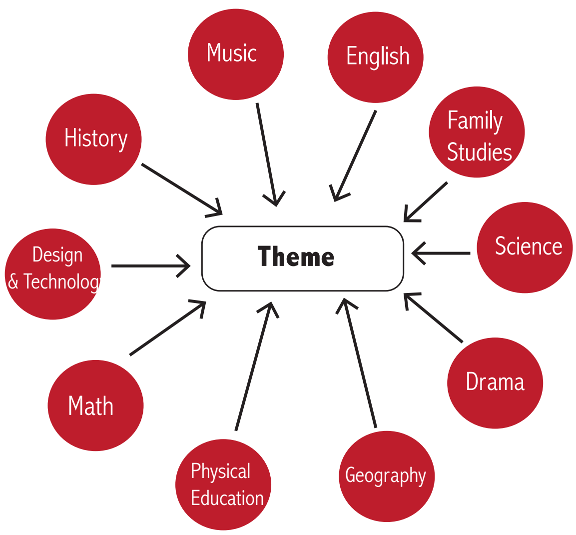
Teachers must develop intriguing curriculum by going beyond the traditional teaching of content based or fragmented teaching to one who is knowledge based and who should be perceived as a 21st century innovative educator. Curriculum integration is a holistic approach to learning thus curriculum integration in PNG SBC will have to equip students with the essential knowledge, skills, values and attitudes that are deemed 21st Century.

There are three approaches that PNG SBC will engage to foster conducive learning for all its children whereby they all can demonstrate proficiency at any point of exit. Adapting these approaches will have an immense impact on the lives of these children thus they can be able to see themselves as catalyst of change for a competitive PNG. Not only that but they will be comparable to the world standards and as global citizens.

Engaging these three approaches in our curriculum will surely sharpen the knowledge and ability of each child who will foresee themselves as assets through their achievements thus contribute meaningfully to their country. They themselves are the agents of change. Integrated learning will bear forth a generation of knowledge based populace who can solve problems and make proper decisions based on evidence. Thus, PNG can achieve its goals like the Medium Term Development Goals (MTDG) and aims such as the Vision 2050 for a happy, healthy and wealthy society whereby, all its citizens should have access and fair distribution to income, shelter, health, education and general good and services improving the general standard of living for PNG in the long run.

1. (i) Multidisciplinary Approach

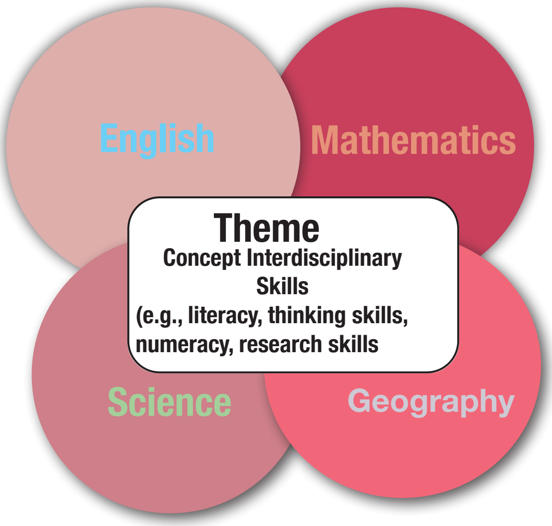
In this approach learning involves a theme or concept that will be taught right across all subject area of study by students. That is, content of a particular theme will be taught right across all subjects as shown in the diagram below. For instance, if the theme is global warming, subject areas create lessons or assessment as per their subjects around this theme. Social Science will address this issue, Science and all other subject likewise.



1. (ii) Interdisciplinary Approach

This approach addresses learning similarly to the multidisciplinary approach of integrated learning whereby learning takes place within the subject area. However, it is termed interdisciplinary in that the core curriculum of learning is interwoven into each subject under study by the students. For instance; in Social Science under the strand of geography students write essay on internal migration however, apart from addressing the issues of this topic, they are to apply the skill of writing text types in their essay such as argumentative essay, informative, explanatory, descriptive, expository and narrative essay while writing their essay. They must be able to capture the mechanics of English skills such as grammar, punctuation and so forth. Though these skills are studied under English they are considered as core skills that cut across all subjects under study. For example; if Science students were to write about human development in biology then the application of writing skills has to be captured by the students in their writing. It is not seen as an English skill but a standard essential skill all students must know and do regardless.

Therefore, essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship in this learning.



2. Intradisciplinary approach

This approach involves teachers integrate sub disciplines within a subject area. For instance, within the subject Social Science, the strands (disciplines) of geography, environment, history, political science and environment will all be captured studying a particular content for Social Science. For example, under global warming, students will study the geographical aspects of global warming, environmental aspect of global warming and likewise for history, political science and economics. Thus, children are well aware of the issues surrounding global warming and can address it confidently at each level of learning.

3. Trans disciplinary Approach

In this approach learning goes beyond the subject area of study. Learning is organized around students' questions and concerns. That is, where there is a need for change to improve lives, students develop their own curriculum to effect these need. The trans-disciplinary approach addresses real-life situations thus giving the opportunity to students to attain real life skills. This learning approach is more to do with Project–Based Learning also referred to as problem-based learning or place- based learning.

The three steps to planning project based curriculum.

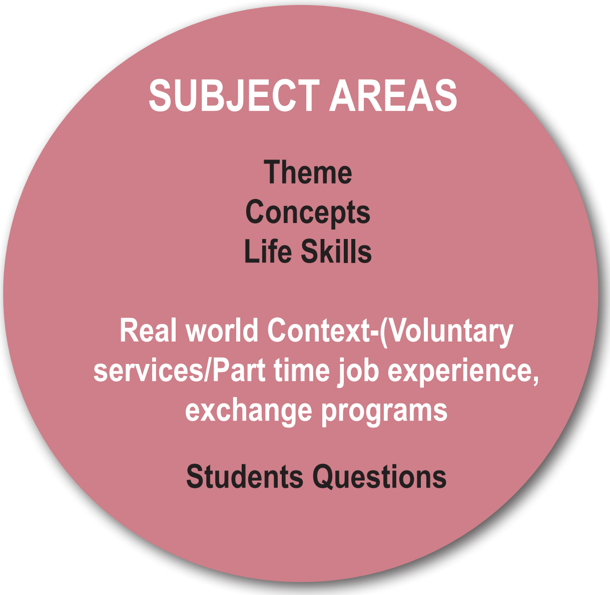
1. Teachers and students select a topic of study based on student interests, curriculum standards, and local resources

2. The teacher finds out what the students already know and helps them generate questions to explore. The teacher also provides resources for students and opportunities to work in the field

3. Students share their work with others in a culminating activity. Students display the results of their exploration and review and evaluate the project.

For instance; students may come up with slogans for school programs such as 'Our culture – clean city for a healthier PNG'. The main aim could be to curb betel nut chewing in public areas especially around bus stops and local markets. Here, students draw up their own instructions and criteria for assessment which is; they have to clean the nearest bus stop or local market once a week throughout the year. They also design and create posters to educate the general public as their program continues. They can also involve the town council and media to assist them especially to carry out awareness.
Studies have proven that Project based-programs have led to the following:

- Students go far beyond the minimum effort
- Make connections among different subject areas to answer open-ended questions
- Retain what they have learnt
- Apply learning to real-life problems
- Have fewer discipline problems
- Lower absenteeism



These integrated learning approaches will demand for teaches to be proactive in order to improve students learning and achievement. In order for PNG Standards-Based Curriculum to serve its purpose fully, these three approaches must be engaged for better learning for the children of PNG now an in the future.

Essential Knowledge, Skills, Values and Attitude and Mathematical Thinking

Students' level of proficiency and progression towards the attainment of content standards will depend on their mastery and application of essential knowledge, skills, values, and attitudes in real life or related situations. Provided here are examples of different types of knowledge, processes, skills, values, and attitudes that all students are expected to learn and master as they progress through the grades. These are expanded and deepen in scope and the level of difficulty and complexity are increased to enable students to study in-depth the subject content as they progress from one grade to the next.

These knowledge, skills, values and attitudes have been integrated into the content standards and benchmarks. They will also be integrated into the performance standards. Teachers are expected to plan and teach essential knowledge, skills, values and attitudes in their lessons, and assess students' performance and proficiency, and progression towards the attainment of content standards.

Types of Knowledge

There are different types of knowledge. These include;	
• Public and private (privileged) knowledge • Specialised knowledge • Good and bad knowledge • Concepts, processes, ideas, skills, values, attitudes • Theory and practice • Fiction and non-fiction • Traditional, modern, and postmodern knowledge	• Subject and discipline-based knowledge • Lived experiences • Evidence and assumptions • Ethics and Morales • Belief systems • Facts and opinions • Wisdom • Research evidence and findings • Solutions to problems

Types of Processes

There are different types of processes. These include;	
• Problem-solving • Logical reasoning • Decision-making • Reflection	• Cyclic processes • Mapping (e.g. concept mapping) • Modelling • Simulating
Mathematics Inquiry processes include: • Gathering information • Analysing information • Evaluating information • Making judgements • Taking actions	

Mathematical Thinking Processes

The five Mathematical process skills that can help the students improve their mathematical thinking.

1. Mathematical Problem Solving

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

2. Mathematical Communication

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments.

3. Mathematical Reasoning

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations.

4. Mathematical Connections

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems
- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding.

5. Mathematical Representation

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

Types of Skills

There are different types of skills. These include:

1. Cognitive (Thinking) Skills

Thinking skills can be categorized into **critical thinking** and **creative thinking** skills.

i. Critical Thinking Skills

A person who thinks critically always evaluates an idea in a systematic manner before accepting or rejecting it. Critical thinking skills include;	
• Attributing • Comparing and contrasting • Grouping and classifying • Sequencing • Prioritising • Analysing	• Detecting bias • Evaluating • Meta-cognition (Thinking about thinking) • Making informed conclusions.

ii. CreativeThinking Skills

A person who thinks creatively has a high level of imagination, able to generate original and innovative ideas, and able to modify ideas and products. Creative thinking skills include;	
• Generating ideas • Deconstruction and reconstruction • Relating • Making inferences • Predicting • Making generalisations • Visualizing	• Synthesising • Making hypothesis • Making analogies • Invention • Transformation • Modelling • Simulating

2. **Reasoning Skills** - Reason is a skill used in making a logical, just, and rational judgment.

3. **Decision-Making Skills** - Decision-making involves selection of the best solution from various alternatives based on specific criteria and evidence to achieve a specific aim.

4. **Problem Solving Skills** – These skills involve finding solutions to challenges or unfamiliar situations or unanticipated difficulties in a systematic manner.

5. Literacy Skills

A strong emphasis must be placed on various types of literacy, from financial to technological, from media to mathematical, from content to cultural. Literacy may be defined as the ability of an individual to use information to function in society, to achieve goals and to develop her or his knowledge and potential. Teachers emphasize certain aspects of literacy over others, depending on the nature of the content and skills they want students to learn.

The following literacy skills are intended to be exemplary rather than defi nitive	
• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Explore complex patterns, interactions and relationships • Differentiate between and among various options	• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Develop an ability to use and apply abstract principals • Explore and/or observe, identify, and analyse how individuals and/or societies relate to one another

6. **High Level Thinking Skills -** These skills include analysis, synthesis, and evaluation skills.

i. Analysis Skills – Analysis skills involve examining in detail and breaking information into parts by identifying motives or causes,underlying assumptions, hidden messages; making inferences and finding evidence to support generalisations, claims, and conclusions.

Keywords				
Analyse	Differences	Find	List	Similar to
Appraise	Discover	Focus	Motivate	Simplify
Arrange	Discriminate	Function	Omit	Take part in
Assumption	Discussion	Group	Order	Test for
Breakdown	Distinction	Highlight	Organize	Theme
Categorize	Distinguish	In-depth	Point out	
Cause & effect	Dissect	Inference	Research	
Choose	Divide	Inspect	See	
Classify	Establish	Isolate	Select	
Comparing	Examine	Investigate	Separate	

ii. Synthesis Skills – Synthesis skills involve changing or creating something new, \ compiling information together in a different way by combining elements in a new pattern proposing alternative solutions.

iii. Evaluation Skills – Evaluation skills involve justifying and presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on set criteria.

Types of Values

Personal engagement and civic engagement strategies help young people to acquire and apply skills and dispositions that will prepare them to become competent and responsible citizens.

1. Personal Values (importance, worth, usefulness, etc.)

Core values	Sustaining values
• Sanctity of life • Truth • Aesthetics • Honesty • Human • Dignity • Rationality • Creativity • Courage • Liberty • Affectivity • Individuality	• Self-esteem • Self-reflection • Self-discipline • Self-cultivation • Principal morality • Self-determination • Openness • Independence • Simplicity • Integrity • Enterprise • Sensitivity • Modesty • Perseverance

2. Social Values

Core values	Sustaining values
• Equality • Kindness • Benevolence • Love • Freedom • Common good • Mutuality • Justice • Trust • Interdependence • Sustainability • Betterment of human kind • Empowerment	• Plurality • Due process of law • Democracy • Freedom and liberty • Common will • Patriotism • Tolerance • Gender equity and social inclusion • Equal opportunities • Culture and civilisation • Heritage • Human rights and responsibilities • Rationality • Sense of belonging • Solidarity • Peace and harmony • Safe and peaceful communities

Types of Attitudes

Attitudes - Ways of thinking and behaving, points of view	
• Optimistic • Participatory • Critical • Creative • Appreciative • Empathetic • Caring and concern • Positive • Confident • Cooperative	• Responsible • Adaptable to change • Open-minded • Diligent • With a desire to learn • With respect for self, life, equality and excellence, evidence, fair play, rule of law, different ways of life, beliefs and opinions, and the environment.

Teaching and Learning Strategies

Mathematics teaching emphasises and embraces the use of cognitive, reasoning, decision-making, problem solving and higher level thinking skills to teach to enhance students' understanding of inter-disciplinary concepts and issues in relation to environment, geography, history, politics and economic within PNG and globally. It aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students must be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

These teaching and learning strategies will help teachers to;

- familiarize themselves with different methods of teaching in the classroom.
- develop an understanding of the role of a teacher for application of various methods in the classroom.

Successful teachers always keep in view that teaching must "be dynamic, challenging and in accordance with the learner's comprehension. He/she does not depend on any single method for making his/her teaching interesting, inspirational and effective.

A detailed table of Teaching and Learning Strategies are outlined below:

STRATEGY	TEACHER	STUDENTS
CASE STUDY Used to extend students' understanding of real life issues.	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE A method used to increase students' interest,involvement and participation.	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/ standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points.
DISCUSSION The purpose of discussion is to educate students about the process of group thinking and collective decision.	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses.	Students ponder over the question and answer by providing ideas, experiences and examples.

STRATEGY	TEACHER	STUDENTS
	Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	
GAMES AND SIMULATIONS Encourages motivation and creates a spirit of competition and challenge to enhance learning.	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win.
OBSERVATION Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.	Give instructions and monitor every activity students do.	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students • Observe and ask essential questions • Record • Interpret.
PEER TEACHING & LEARNING (power point presentations, pair learning) Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching.	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class.

STRATEGY	TEACHER	STUDENTS
PERFORMANCE-RELATED TASKS (dramatization, song/lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date.	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria.	Go into their established working groups. Being creative and create dramas, songs/lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively.	Teacher outline the steps and procedures of how to do and the criteria.	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students.	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing.	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

Strands, Units, Topics and Suggested Lesson Titles

This section contains the overview of Mathematics content to be taught in grade 9. The table below outlines strands, units,topics with suggested lesson titles. Teachers will use this to develop their own termly and yearly programs.

STRAND	UNIT	TOPIC	SUGGESTED LESSON TITLES
Number, Operation and Computation	Number and operations	Factors, Multiples, Primes and Composites	• Review of Factors, Multiples, Primes and Composites • Prime Factors • Highest Common Factor (HCF) • Lowest Common Multiple (LCM)
			• Primes and Composites • Exponents (Squares and Cubes) • Fractions, Decimals and Percentages • Expressing Fractions and Percentage as Decimals and vice versa • Expressing one quantity as a percentage of another • Comparing two Quantities by Percentage • Increasing and Decreasing a Quantity by a given Percentage
		Ratio And Proportion	• Simplifying Ratios • Fractional ratios • Proportional Parts • Direct Proportion • Inverse Proportion
	Estimation	Approximation and Estimation	• Rounding off Quantities • Estimating Percentage of Quantities • Estimating a Quantity as a Percentage • Word Problems on Estimation
		Significant Figures	• Significant Figures and Decimals • Significant Figures and Decimals • Problem Solving on Significant Figures • Problem Solving on Decimals
	Directed Numbers	Directed Numbers	• Rules of Directed Numbers (Addition & Subtraction) • Rules of Directed Numbers (Multiplication & Division) • Rules of Mixed Operations • Solve everyday problems with Directed Numbers • Solve everyday problems using Mixed Operations
		Representation and Application of Directed Numbers	• Absolute Value on a Number Line • Directed Numbers using real-life situations. • •
	Indices	Squares and Square Roots	• Square Numbers • Square Roots • Problem Solving involving Square Roots
		Real Numbers Introduction of the real Number System	• Integers • Rational and Irrational Numbers • Evaluating Expressions involving Index Notation • Compare and order rational numbers and square roots using a number line • Scientific Notation

	Indices	Operations with Rational Numbers	• Addition and Subtraction • Multiplication and Division • Order of Operations
		Laws of Indices	• Positive Indices • Negative Indices • Zero Indices • Powers of Powers • Summary of Laws of Indices • Solve problems using the Index Laws
Geometry, Measurement and Transformation	Mensuration of Plane figures and Solids	Metric System of Measurement	• Metric systems of measurement • Estimating Length
		Perimeter	• Perimeter of Plane Figures • Circumference of a Circle and Sector • Perimeter of composite shapes
		Area of Plane Figures	• Area of Triangles and Quadrilaterals • Area of Circles and Sectors • Area of Composite Shapes
		Surface Area and Volume	• Nets and Surface Area of Prisms • Nets and Surface Area of Cylinders • Nets and Surface Area of Pyramids
			• Volume of Prisms • Volume of Cylinders • Volume of Pyramids • Surface area and volume of; • right pyramids, • right cones, • spheres and • related composite solids
	Rates	Time Scales	• Time and Rates • Time scales and intervals
		Rate, Tables and Graphs	• Measure of Rates • Rates, tables and graphs
		Travel Graphs	• Distance – Time Graphs • Speed- Time Graphs • Practical applications of rates of change

	Rates	Angles and Lines	• Angles formed by Parallel lines and a transversal • Perpendicular lines • Angle sum of polygons
	Geometry	Transformations including Reflections, Rotation and Translation	• Types of Transformation of shapes • Reflection • Rotation • Translation
		Congruency and Similarity	• Measures of congruency • Properties of similarity • Congruency and similarity of plane figures
		Circles	• Parts of a circle • Angles formed by chords • Angles formed by Tangent • Review
Patterns and Algebra	Linear Functions	Equation of a straight line	• Gradient of a straight line ($y = mx+c$) • Equation given the gradient and y-intercept • Equation given two points • Equation given one point and the gradient • Equation given the graph
		Graphs of Linear equation	• Solve Linear equations • Plotting and sketching linear equations
		Linear Relations	• Interpret linear relationship between two quantities • Graph the linear relationship between two quantities
	Equations and inequality	Linear simultaneous equations	• Linear equations • Solving simultaneous equations using substitution method • Solving simultaneous equations using elimination method • Solving simultaneous equations using graphs and digital technology • Solve word problems involving simultaneous equations
		Linear equations and inequalities	• Basic rules for solving inequalities • Solving Inequalities with single variables • Representing Inequalities on a Number Line • Solve Linear Inequalities and represent the solutions set on a number line.
			Review

	Patterns, sequences and formulae	Patterns, Sequences and Formulae	• Completing and describing number patterns • Finding terms and writing rules for sequences • Using the T_n formula
		Formulae and Patterns	• Identify formulae for various patterns • Apply formulae to find values of various patterns • Number pattern problems • Solve problems of various patterns
	Algebra	Algebraic Equations and Factorization	• Formulate Algebraic expressions • Formulate Algebraic equations • Factorise algebraic expressions
		Factorisation of Monic and Non-monic Quadratic Expressions	• Factorise Monic quadratic expressions • Factorise non-monic quadratic expressions • Solve quadratic equations
		Expansion of Binomial Products	• Simplify algebraic expressions • Algebraic products • Algebraic quotients
		Algebraic Fractions	• Addition and subtraction of algebraic fractions • Multiplication and division of algebraic fractions • Word problems on algebraic fractions • Review
Statistics and Orobability	Data and Statistics	Data	• Types of data (Discrete and Continues Variables) • Calculation of Mean, Mode, Median and range • Analysing & Reporting
		Exploring data	• Stem and leaf plots • Comparing sets of data • Data collection, analysis and Representations • Plan and design data collection method • Data collection • Organization of data • Representation of data
	Probability • Events using 'at least' • Exclusive events • Inclusive events	Experimental and Theoretical Probabilities	• Introduction to probability • Probability based on experiments • Probability based on theory • Representation of Probability using Tree Diagrams
		Complementary events	• Define and identify complementary events • Problem solving using sum of probabilities
		Language of Probability	• Events using 'at least' • Exclusive events • Inclusive events

Grade 9
Mathematics
Teaching Content

Strand 1: Number, Operations and Computation

Content Standard: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as the key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Units	Benchmark	Topics	Lesson Titles
Number and Operations	**9.1.1.1** Use and represent factors, multiples, primes, composites, in various forms and situations.	Factors, Multiples, Primes and Composites	• Review of Factors, Multiples, Primes and Composites • Prime Factors • Highest Common Factor (HCF) • Lowest Common Multiple (LCM) • Primes and Composites • Exponents (Squares and Cubes)
	9.1.1.2 Calculate fractions, decimals and percentages and their conversions.	Fractions, Decimals and Percentages	• Expressing Fractions and Percentage as Decimals and vice versa • Expressing one quantity as a percentage of another • Comparing two Quantities by Percentage • Increasing and Decreasing a Quantity by a given Percentage
	9.1.1.3 Solve problems involving ratio and proportions between quantities.	Ratio and Proportion	• Simplifying Ratios • Fractional ratios • Proportional Parts • Direct Proportion • Inverse Proportion
Estimation	**9.1.1.4** Estimate a reasonable solution to a problem using rounding and estimation and recognize their limitations through calculation methods.	Approximation and Estimation	• Rounding off Quantities • Estimating Percentage of Quantities • Estimating a Quantity as a Percentage • Word Problems on Estimation
	9.1.1.5 Identify significant figures and decimals in various situations.	Significant Figures	• Significant Figures and Decimals • Significant Figures and Decimals • Problem Solving on Significant Figures • Problem Solving on Decimals
Directed Numbers	**9.1.1.6** Use appropriated rules of directed number and mixed operations to solve authentic problems.	Directed Numbers	• Rules of Directed Numbers • (Addition & Subtraction) • Rules of Directed Numbers (Multiplication & Division) • Rules of Mixed Operations • Solve everyday problems with Directed Numbers • Solve everyday problems using Mixed • Operations
	9.1.1.7 Explain the concepts of negative and positive numbers using various representation.	Representation and Application of Directed Numbers	• Absolute Value on a Number Line • Directed Numbers using real-life situations.

Indices	**9.1.1.8** Determine the relationship between square numbers and square roots and solve related problems.	Squares and Square Roots	• Square Numbers • Square Roots • Problem Solving involving Square Roots
	9.1.1.9 Explain the rational and irrational numbers and perform operations with surds and fractional indices.	Real Numbers	• Introduction of the real Number System • Integers • Rational and Irrational Numbers • Evaluating Expressions involving Index Notation • Compare and order rational numbers and square roots using a number line • Scientific Notation
	9.1.1.10 Apply addition, subtraction, multiplication, division and order of Operations when calculating with rational numbers.	Operations with Rational Numbers	• Addition and Subtraction • Multiplication and Division • Order of Operations
	9.1.1.11 Apply the index laws to variables, using positive integer indices and the zero.	Laws of Indices	• Positive Indices • Negative Indices • Zero Indices • Powers of Powers • Summary of Laws of Indices • Solve problems using the Index Laws

Unit: Number and Operations	Topic: Factors, Multiples, Primes and Composites
Benchmark	**9.1.1.1** Use and represent factors, multiples, primes, composites, in various forms and situations.

Learning Objectives: By the end of the topic, students will be able to;

- define and explain the properties of Factors, Multiples, Primes and Composite Numbers,
- revise how to identify Highest and Lowest Common Factors, and
- identify Square Numbers and Cubes.

Essential questions:

- What are the important properties of Factors, Multiples, Primes and Composites?
- How can prime factors be identified?
- What must we do to find the Highest Common Factor (HCF) and Lowest Common Multiplier (LCM)?
- How can we differentiate between primes and composites?
- What happens when a number is squared or cubed?

Key Concepts (ASK-MT)	
Attitudes/Values	• Develop interest and appreciate the properties of numbers. • Think independently in identifying various types of numbers.
Skills	• Identify the properties of Factors, Multiples, Primes and Composite Numbers and use to revise Highest and Lowest Common Factors and to calculate and represent in various forms.
Knowledge	Develop understanding of the Properties of: • Factors, Multiples, Primes and Composites. • Prime Factors. • Highest Common Factor(HCF). • Lowest Common Multiple(LCM). • Exponents.
Mathematical Thinking	• Think about how to identify and express key features or Properties of Numbers.

Content Background

Factors & Multiples

When an integer p is divisible by a non-zero integer q, that is p = q x (integer), we say that q is a factor (divisor) of p and that p is a multiple of q.

Example: Since 56 = 7 x 8,
7 and 8 are factors of 56, and 56 is a multiple of 7 and 8.

Primes and Composites

Numbers which have only two factors which are 1 and itself are prime numbers.
47 is a prime number since it only has two factors, 1 and 47.
An integer which is non - prime is called a composite number.
E.g. 28 is a composite number because 28 = 4 x 7

The factors of 12 are 1, 2, 3, 4, 6 and 12. In all the factors listed, 2 and 3 are prime numbers. They are called the prime factors.

Squares and Cubes

If a number is multiplied by itself, it is said to be squared. The square of a whole number is a perfect square.

Example: 5 x 5 = 25. This can be written as $5^2 = 25$. 25 is a perfect square.

Similarly, 43 means, 4 x 4 x 4 which is 64 and can be written as $4^3 = 64$. We say that the cube of 4 is 64.

Unit: Number and Operations	Topic: Fractions, Decimals and Percentages
Benchmark	**9.1.1.2** Calculate fractions, decimals and percentages and their conversions.

Learning Objectives: By the end of the topic, students will be able to;

- calculate operations with Fractions, Decimals and Percentages,
- convert Decimals to Fractions to Percentages and vice versa, and
- apply Fractions, Decimals and Percentages in solving problems in daily life.

Essential questions:

- What are the rules of Adding, subtracting, Multiplying and Dividing fractions and decimals?
- How are fractions and decimals converted to percentages?
- How is a percentage converted to fractions and decimals?
- How can we calculate the percentage of a quantity?
- What is the method of calculating percentage change?

Key Concepts (ASK-MT)	
Attitudes/Values	• Show confidence in applying Mathematical knowledge in daily life. • Show sensitivity towards the importance of Mathematics in their daily lives.
Skills	• Perform numerical computations with fractions decimals and percentages, compare quantities by percentage and calculate mentally and also provide quick estimates of the accuracy of calculations.
Knowledge	• Operations with Fractions and Decimals. • Conversion of Percentages to Fractions and Decimals and vice versa. • Percentage Change.
Mathematical Thinking	• Think about how to identify and express the relations of fractions decimals and percentages and choose appropriate methods to solve problems .

Content Background

1. Conversion between Decimals and Fractions

It should be known by now that decimals are fractions with denominators of 10, 100, 1000, etc., determined by the decimal places.

Example: $0.53 = \frac{53}{100}$ and $0.625 = \frac{625}{1000}$.

2. Fractions and Decimals to Percentage

To convert a fraction to percentage, multiply by 100.

Example: (a) $\frac{17}{20} = \frac{17}{20} \times 100 = 85\%$ (b) $0.3 = 0.3 \times 100 = 30\%$

3. Percentage to Fractions and Decimals

To change a percentage into fractions, divide by 100.

Example: (a) $15\frac{1}{2}\% = 15\frac{1}{2} \div 100 = \frac{31}{200}$

(b) $9.2\% = 9.2 \div 100 = 0.092$

4. Percentage of a Quantity

To find the percentage of a quantity, we can convert the percentage to a decimal and calculate.

Example: (a) 10% of K60 = 0.1×60 = K6

(b) $8\frac{1}{4}\%$ of 65 m = $0.0825 \times 65 = 5.3625$ m

5. Percentage Change

When a quantity increases or decreases, the increase or decrease can be expressed as a percentage of the original value using the following:

Percentage Increase = $\frac{\textit{Increase}}{\textit{Original Value}}$ x 100%

Percentage decrease = $\frac{\textit{Decrease}}{\textit{Original Value}}$ x 100%

Example

a. Increase 100 by 30%   b. decrease 320 by 25%

Solution

a. $\frac{30}{100} \times 100 = 30, \quad \therefore 30 + 100 = 130$

a. $\frac{25}{100} \times 320 = 80, \quad \therefore 320 - 80 = 240$

Unit: Number and Operations	Topic: Ratio and Proportion
Benchmark	**9.1.1.3** Solve problems involving ratio and proportions between quantities.

Learning Objectives: By the end of the topic, students will be able to;

- simplify Ratios and Fractional Ratios,
- calculate quantities using a given ratio, and
- define and solve problems involving ratio and direct or inverse proportionality of two quantities.

Essential questions:

- How is a fractional ratio simplified?
- What is a proportional relationship?
- How can we describe direct proportion and inverse proportion?

Key Concepts (ASK-MT)	
Attitudes/Values	• Think independently in solving various types of problems. • Show confidence in applying mathematical knowledge in daily life.
Skills	• Express quantities as ratios. • Simplify ratio and fractional ratios. • Calculate the amount of proportional parts. • Solve problems of quantities of ratios.
Knowledge	• Ratio and fractional ratios. • Proportional parts. • Direct proportion and Inverse proportion.
Mathematical Thinking	• Think about and choose appropriate methods to solve various problems and summarize the main concepts of ratio and proportional relations.

Content Background

1. Fractional Ratios

Ratios can be expressed as fractions where a : b is expressed as $\frac{a}{b}$. In order to simplify fractional ratios, we multiply each quantity by the LCM of their denominators.

Example (a) $\frac{1}{2} : \frac{2}{3}$ is simplified by multiplying by 6.

$\frac{1}{2} \times 6 : \frac{2}{3} \times 6$

3 : 4

(b) In the case of $1\frac{1}{2} : \frac{3}{4}$, we convert $1\frac{1}{2}$ to $\frac{3}{2}$ and multiply $\frac{3}{2} \times 4 : \frac{3}{4} \times 4$ (LCM) = 2 : 1

2. Directly proportional; as one amount increases, another amount increases at the same rate.

Two quantities are said to be in direct proportion when:

(a) One quantity increases 2, 3… times as much, or decreases $\frac{1}{2}$, $\frac{1}{3}$,…, the other quantity also increases 2, 3,…, times as much, or decreases $\frac{1}{2}$, $\frac{1}{3}$,…., respectively.
Example:

(b) One quantity becomes m times as much, the other quantity also becomes m times as much.

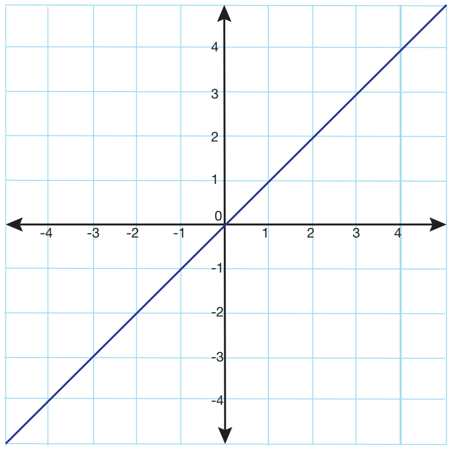
(c) The ratio of the two quantities (ratio) is constant.

The symbol for 'directly proportional' is $\propto$

Example 1: You are paid K3 per hour
How much you earn is directly proportional to how much hours you work.
Work more hours, get more pay; in direct proportion.
This can be written as Earnings $\propto$ hours worked.

Earnings = K3 x hours worked
This can be written as $y = kx$, where k is the constant proportionality of y, and y is directly proportional to x.
When $y = 24$ and $x = 8$,
Then $24 = k \times 8$, $k = 3$
$y = 3x$, so constantly proportionality of y is 3

Example 2: What is the value of y when x = 15 and k is 3
$y = 3 \times 15$
$y = 45$,
You will earn K45, if you worked for 15 hours
The graph of the direct proportion is as represented below;



3. Inversely proportional: as one amount **decreases,** another amount increases at the same rate.

Two quantities are said to be in Inverse Proportion when:
(a) One quantity increases 2, 3, ... times as much, or decreases , $\frac{1}{2}$, $\frac{1}{3}$,…, the other quantity becomes $\frac{1}{2}$, $\frac{1}{3}$,…, times as much or 2, 3, ... times as much, respectively.
(b) One quantity becomes m times as much; the other quantity becomes $\frac{1}{m}$ times as much.
The product of the two quantities remains constant. The range of numbers for the quantities is limited to non-negative numbers only.

Example 1: Speed and travel time
Speed and travel time are inversely proportional because the faster we travel the travel time goes down and when the speed goes down the travel time goes up.
In the formula, *Speed* (S) = $\frac{Distance\ (d)}{Time\ (t)}$, the equation can be written as $y = \frac{k}{x}$
Where k is constant proportion, therefore y is inversely proportional to x ,which is the same as y is directly proportional to $\frac{1}{x}$.

Example 2 : 4 people can plough the gardening soil of 100 m^2 for 3 hours
How long will it take for 6 people to plough the gardening soil of 100 m^2?
We can use $t = \frac{k}{n}$, where t= number of hours, k = constant proportionality, n = number of people
When t = 3, and n = 4 , then $3 = \frac{k}{4}$, k = 12, so it will take 2 hours for 6 people to plough the 100 m^2.

How many people are needed to complete the job in only half an hour?

This can be represented on a table below and can also be represented on the line graph.

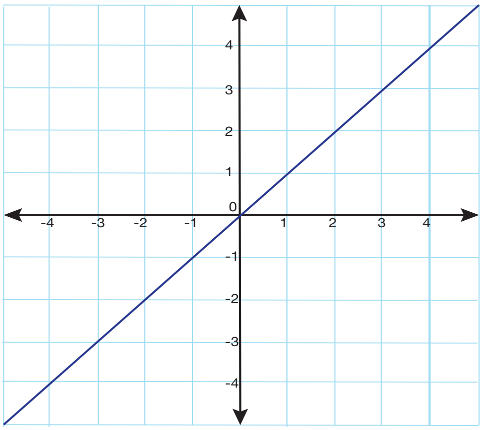
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t	*12*	*6*	*4*	*3*	*2.4*	*2*	*1.7*	*1.5*	*1.3*	*1.2*	*1.1*	*1*	*0.9*											*0.5*

n Summary the line graphs of direct proportional and inversely proportional should be represented as such in the below;

Directly Proportional

$y = \propto x$

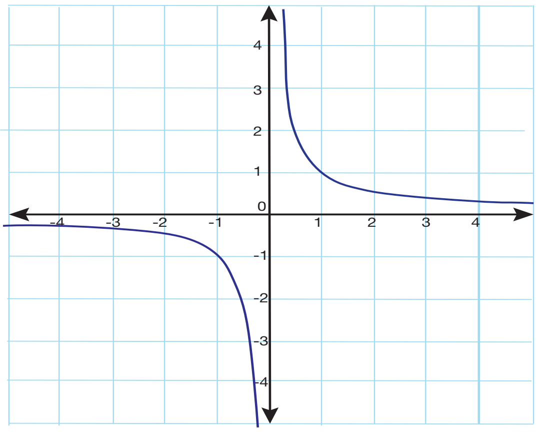
$y = kx$ for a constant k



Inversely Proportional

$y = \propto x$

$y = \frac{k}{x}$ for a constant k



Unit: Estimation	Topic: Approximation and Estimation
Benchmark	**9.1.1.4** Estimate a reasonable solution to a problem using rounding and estimation and recognize their limitations through calculation methods.

Learning Objectives: By the end of the topic, students will be able to;

- determine the approximate value of quantities by estimation, and
- prove answers by calculation.

Essential questions:

- What are some ways of approximation and estimation of decimal quantities?

Key Concepts (ASK-MT)	
Attitudes/Values	• Show sensitivity towards the importance of Mathematics in their daily lives.
Skills	• Estimate and round off approximate values, calculate and determine their numerical computations.
Knowledge	• Rounding and Estimating Quantities. • Mental Calculation.
Mathematical Thinking	• Think about how to make reasonable estimations with proof of answers.

Content Background

1. Approximations and Estimation

The number of decimal places corresponds to its number of digits after the decimal point. For example, 0.56 has 2 decimal places and 32.487, has 3 decimal places. When rounding off a number to 1 decimal place, we look at the digit in the 2nd decimal place and round up if it is greater than or equal to 5 and round down if it is less than 5. When rounding off to 2 decimal places, we look at the digit in the 3rd decimal place and round up or round down and so on.

Using this method of rounding, we can be able to estimate to make sure that calculations are sensible. Students should be encouraged to choose numbers which are easy to add subtract and multiply. If division is needed, select numbers which will cancel or divide out exactly.

Example: Multiply 32.4 by 0.259

(i) For a rough estimate we use $32 \times 0.25 = 32 \div 4$
$= 8$

(ii) Accurate calculation $32.4 \times 0.259 = 8.39$ (rounded to 2 d.p)

The rough estimate shows that the answer is sensible.
We can also estimate the numbers in percentages for real life situations.

For example; Give estimate percentages for the following problems

- What will be the percentage of grade 9 students from the population of grades from 7 to 10 students? ~20%
- Estimate the percentage of female teachers teaching in your school e.g. ~25%
- Two boys painted the wall of the classroom with 2 tins of paint, but could not complete the wall. It was estimated that they needed one more paint to complete the task. What was the percentage of the painted wall? ~ 75%
- How much of water in percentages would you use to cook two cups of rice? ~ 80%
- Egg cartons are made entirely from recycled paper. What percentage of the carton comes from recycled paper? ~ 100%

Unit: Estimation	Topic: Significant Figures
Benchmark	**9.1.1.5** Identify significant figures and decimals in various situations.

Learning Objectives: By the end of the topic, students will be able to;

- round numbers and measures to an appropriate degree of accuracy to determine its significant figure, and
- solve problems involving Significant Figures and Decimals in various situations.

Essential questions:

- What is an approximate value?
- What are significant figures?
- How can we determine significant figures?

Key Concepts (ASK-MT)	
Attitudes/Values	• Think independently in solving various types of problems. • Appreciate the value of Significant Figures.
Skills	• Identify approximate value in decimal numbers and round off. • Calculate and determine their numerical computation.
Knowledge	• Concept of Significant Figures. • Approximate Value of decimal figures.
Mathematical Thinking	• Think about how to make reasonable estimation of significant figures with proof of answers.

Content Background

1. Significant Figures (sf)

Significant figures (sf) of a number are digits that carry meaning contributing to its measurement resolution. Significant arithmetic is approximate rules for roughly maintaining significance throughout computation. There are more sophisticated scientific rules one of which is known as propagation or uncertainty but simple significant arithmetic will be commonly use.

The following rules are used to determine the number of Significant Figures;

- All non-zero digits are significant whenever they are written down.
  Example: 1.23 has 3 significant figures
- Zeros that lie between non-zero digits are significant
  Example: 20.01 has 4 significant figures
- Zeros which are not preceded by non-zero digits are not significant
  Example: 0.0012 has 2 significant figures
- The final zeros which appear after the decimal points are significant
  Example: 1.00 has 3 significant figures
  4.50 has 3 significant figures
  0.10 has 2 significant figures
- The zeros in whole numbers may or may not be significant depending on the estimations made.
  Example: 2001 = 2000 (correct to 3 significant figures)
  2001 = 2000 (correct to 2 significant figures)
  2001 = 2000 (correct to 1 significant figure)

Example 1:
Numbers are often rounded to avoid reporting insignificant figures. For example, it would create false precision to express a measurement such as 12.34525 kg (which has 7 significant figures) if the scales only measured to the nearest gram and gave a reading of 12.345 kg (which has 5 significant figures). Numbers can also be rounded merely for simplicity rather than to indicate a given precision or measurement, for example to make them faster to pronounce in news broadcasts.
Round off the following numbers to two figures

1. 1539 = 1.539 = 1.5×10^2 2) 0.00736 = 7.36 = 7.4×10^{-3}

Round off 63.75091 to;
i) 1 significant figure (sf) ii) 2 sf iii) 3 sf iv) 4 sf v) 5 sf

2. Determine how many significant figures is in each of the measurements
1) 0.0034050 L = 5 2) 33.600 m = 5 3) 7500.0 g = 5 4) 47,900 mm = 3
5) 7,000,000,001 km =10 6) 8.07 Hz = 3

3. Identify the digits that are not significant in these numbers
1) 7300 2) 0.065 3) 69000 4) 3020.40 5) 8002

4. When a number was rounded off to 2 significant figures, the answer was;
a) 430 b) 3.7

i) What is the smallest the number could have been?
ii) What is the largest the number could have been?
iii) Write a mathematical sentence that shows the range of possible numbers?

a) The second significant number is in the tens column, hence the number has been rounded off to the nearest 10.
i) 425 is halfway between 420 and 430 but by convention, is rounded up to 430. This is the smallest the number could have been
ii) We cannot specify the largest number but we do know that it has to be less than 435 (because 435 can be rounded off to 440)
iii) The number could be equal to 425 or between 425 and 435. We write this as $425 \le \text{number} < 435$

b) The second significant figure is in the first column after the decimal point hence the number has been rounded off to decimal place.
i) 3.65 is half way between 3.6 and 3.7 but by convention is rounded up to 3.7. This is the smallest the number could have been
ii) We cannot specify the largest number but we do know that it has to be less than 3.75 (because 3.75 can be rounded off to 3.8)
iii) The number could be equal to 3.65 or between 3.65 and 3.75.
We write this as $3.65 \le \text{number} < 3.75$

Unit: Directed Numbers	Topic: Directed Numbers
Benchmark	**9.1.1.6** Use appropriated rules of directed number and mixed operations to solve authentic problems.

Learning Objectives: By the end of the topic, students will be able to;

- review the rules of Directed Numbers and its application in the four arithmetic operations, and
- solve problems using the order of operations with directed numbers.

Essential questions:

- What are the rules of directed numbers in addition and subtraction?
- What are the rules of directed numbers in multiplication and division?
- Is the order of operations applicable in dealing with directed numbers?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate the value of Directed Numbers. • Show confidence in solving mathematical problems on directed numbers using the order of operations.
Skills	• Review rules of Directed Numbers, its applications using the four operations and solve problems.
Knowledge	• Rules of Directed Numbers. • Order of Operations.
Mathematical Thinking	• Think about how to draw simple and logical reasons to justify mathematical concepts of Directed Numbers.

Content Background

Rules of Directed Numbers

1. Adding Directed Numbers

- When adding two positive numbers, add the numbers and place a plus sign in front of the sum. e.g (+3) + (+4) = 7 or +7
- When adding two negative numbers, add the numbers and place a negative sign in front of the sum. e.g (-2) + (-3) = -5
- When the augend and addend have two different signs, find the difference of the two numbers and place the sign of the larger number in front of the sum. e.g 3 + (-8) = -5
- If there are more than two numbers, add the positive numbers together and add the negative numbers together. The set of numbers is then reduced to two numbers, one positive and the other negative. We can then proceed on to finding the sum.

2. Subtracting Directed Numbers

When subtracting directed numbers, change the sign of the subtrahend and add (apply rules of addition). Example: 5 - (-3) = 5 + 3 = 8

3. Multiplication

(a) Numbers with like signs multiplied give a positive product.
Example: (a) 3 x 5 = 15 (b) (-3) x (-5) = 15
(b) Numbers with unlike signs multiplied give a negative product.
Example: (-3) x 5 = -15

4. Division

Numbers with like signs divided give a positive quotient.
Example: (a) 20 ÷ 4 = 5 (b) -20 ÷ - 5= 4
Numbers with unlike signs divided give a negative quotient.
Example: (a) -20 ÷ 4 = -5
Example: Simplify
a. (-3) x (-5) x 2 b. 54÷ (-9) x (-2)
Solution
a. (-3) x (-5) x 2 = (-3 x -5) x 2 = 15 x 2 = 30
b. 54÷ (-9) x (-2) = (54 ÷ -9) x -2 = -6 x -2 = 12

Basic Sign Rules
i) + + = +
ii) - + = -
iii) - - = +

5. Adding and subtraction of polynomials (collect like terms to add and subtract)

a. 3x + 2x = 5x b. -3x + 5x = 2x c. -2x + -3x = -5x d. 3x - (-2x) = 5x
e. -3x - 5x = -8x f. 2x + 3-2x = 3 g. 2y - 3y + 2x = -y + 2x h. 5y - 2 = 5y - 2

6. Multiplication and division of polynomials (multiply and divide with numbers and use rules for like terms)

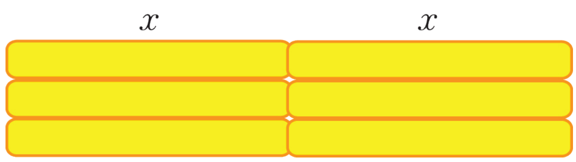
a. -3x x 3 = -9x b. 2x x 3y x 5 = 30xy c. -3x x 2y ÷ 3x = -x x 2y= -2xy
d. 54 ÷ -9x x – 2 = (54 ÷ -9x) x -2 = -6x x -2 = 12x

7. Examples of mixed operations (multiply/divide then add/subtract)

a. -2x + 3x x 2 = -2x + (3x x 2) = -2x x 6x = -12x
b. 3x + 2x - 2y x 3y = 5x-6y c. -2x ÷ 2x x -3x = -x x -3x = $3x^2$
d. $\frac{(6y + 3y)}{2y} = \frac{9y}{2y} = 4.5$ e. $\frac{(6y + 3y^2)}{2y} = \frac{6y}{2y} + \frac{(3y^2)}{2y} = 3+1.5y$

8. Word Problems

a) A PMV Travelling to the village made about 3 stops before it reached its destination of 36 km.
The expression written is 6x + 3x + 3x = 12x, when x = 3, what was the distance in km at the first stop? (18 km)
b) A mother served 6 plates of rice with greens and 2 bananas each 6 (x + 2y) = 6x + 12y
c) Divide 6 tiles in 3 rows; can be written as 6x ÷ 3 = 2x means two x tiles in each row



Unit: Directed Numbers	Topic: Representation and Application of Directed Numbers
Benchmark	**9.1.1.7** Explain the concepts of negative and positive numbers using various representations.

Learning Objectives: By the end of the topic, students will be able to;

- represent and explain the Absolute Value on a Number Line, and
- apply and Use Directed Numbers using real-life situations.

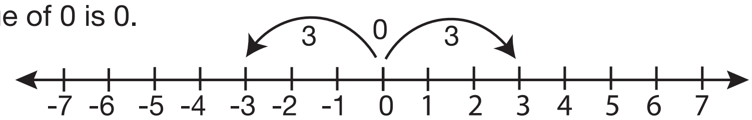
Essential questions:

- What is an absolute value?
- How can we represent the Absolute Value on a number line?
- What are some applicable situations which can be represented by directed numbers?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate the aspects of Directed Number and its uses in daily life. • Think independently in representing Directed Numbers in various ways.
Skills	• Drawing and determining the absolute value on number lines • Represent directed numbers in situations such as: Temperature, Altitude and Depth and Profit/Loss.
Knowledge	• Directed Numbers on the Number line. • Absolute Value.
Mathematical Thinking	• Think about how to identify and express main ideas of Directed Numbers and understand the meaning of the absolute value.

Content Background

What is an absolute value?
In mathematics the absolute value or module [x] of a real number x is the non-negative value of x without regard to its sign. Namely [x] = x for a positive x and [x] = -x for a negative x (in which case -x is positive), and [0] = 0. For example the absolute value of 3 is 3 and the absolute value of -3 is 3. The absolute value of a number may be thought of as distance from zero.
Absolute Value on number Line
We use directed numbers to represent quantities on different sides of a zero reference point, such as, distances above and below sea level or profit and loss of a product.
The distance from the origin (0) to the point which represents a particular number is called the absolute value of it.
Example: On the number line below, the absolute values of both +3 and -3 are both 3 and the absolute value of 0 is 0.



Directed Number using real-life situations.
Determine the absolute values of;

1) Temperature in a very cold climate of Japan was 3 degrees Celsius in December 2015 (3)
2) Altitude – Mt Wilhelm is about 4,509 meters above sea level (4,509)
3) Profit – making profit by selling second hand clothes when demand is high in the village K100 per day in 5 days: Made 50% profit (50)
4) Loss – making loss by selling coconuts when many sellers are also selling coconuts at the same market. E.g. selling at 50^t from K1.50 and K1 from K2 (2)
5) Win – PNG Won their basketball game to Samoa from 30 to 25 in the South Pacific Games (5)
6) Lost – PNG lost their game of football to Vanuatu of 4 to 6 at the south pacific games (2)

Unit: Indices	Topic: Squares and Square Roots
Benchmark	**9.1.1.8** Determine the relationship between square numbers and square roots and solve related problems.

Learning Objectives: By the end of the topic, students will be able to;

- relate square numbers to square roots,
- calculate to find the square root of products, and
- solve problems that involve square numbers and square roots.

Essential questions:

- What is an absolute value?
- How can we represent the Absolute Value on a number line?
- What are some applicable situations which can be represented by directed numbers?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate the aspects of Squares, Square Roots and its uses in daily life. • Show confidence in applying mathematical knowledge solving problems.
Skills	• Perform numerical computations on numbers raised to the power of 2. • Identify and calculate the square root of numbers and products. • Solve problems involving square roots.
Knowledge	• Square Numbers and Square roots.
Mathematical Thinking	• Think about how to distinguish and understand the relationship between square numbers and square roots.

Content Background

1. Square Roots

If a number is multiplied by itself, it is said to be squared. The square of a whole number is a perfect square.

Example: 5 x 5 = 25. This can be written as $5^2 = 25$. 25 is a perfect square.

The square root of a number is the number whose square equals the given number.

Example: Since $5^2 = 25$, the square root of 25 is 5.

The sign $\sqrt{}$ is used to denote a positive square root and we write $\sqrt{25} = 5$.

In the case where we write $(-3)^2 = 9$ and $(+3)^2 = 9$, we write $\sqrt{9} = \pm 3$

2. Square Root of a Product

The square root of a product is the product of square roots of the individual numbers

Example: (a) The square root of 4 x 9: $\sqrt{(4 \times 9)} = \sqrt{4} \times \sqrt{9}$

$= 2 \times 3$

$= 6$

(b) The square root of 16 x 36 x 81: $\sqrt{(16 \times 36 \times 81)} = \sqrt{16} \times \sqrt{36} \times \sqrt{81}$

$= 4 \times 6 \times 9$

$= 216$

3. Problems of square numbers and square roots

Ranu's flower garden is now a square. If he enlarges it by increasing the width 1 meter and the length 3 meters, the area will be 19 square meters more than the present area. What is the length of the side now? A = L x W

$= (x \times x)\ m^2$

$= x^2$

Enlarge:

Length = $x + 3$, width = $x+1$, $x^2 + 19 = (x + 3) \times (x+1)$

Solution

A = l x w

$x^2 + 19 = (x + 3) \times (x + 1)$

$x^2 + 19 = x^2 + 4x + 3$

$16 = 4x$

$4 = x$

Therefore, new length = 7 cm, new width = 5 cm

Unit: Indices | **Topic: Real Numbers**

Benchmark	**9.1.1.9** Explain the rational and irrational numbers and perform operations with surds and fractional indices.

Learning Objectives: By the end of the topic, students will be able to;

- define real numbers and distinguish between rational and irrational numbers,
- perform numerical computation with surds and fractional indices,
- represent rational numbers and square roots on a number line, and
- represent numbers using scientific notation.

Essential questions:

- What is the real number system?
- What is the difference between rational and irrational numbers?
- How are numbers written in index notation?
- What is a scientific Notation?

Key Concepts (ASK-MT)	
Attitudes/Values	• Think independently in solving various types of problems. • Develop Interest and appreciate the properties of the Real Number System.
Skills	• Identifying the properties of real numbers. • Performing numerical computations with surds and fractional indices. • Evaluating expressions using index notation. • Represent rational numbers and square roots on a number line. • Represent numbers using scientific notation.
Knowledge	• Real Number System, Integers, Rational Numbers, Index Notation. • Square Roots, Scientific Notation.
Mathematical Thinking	• Think about how to identify and express the main ideas of Real Numbers, Surds and fractional indices, and summarize key points about the properties of Real Numbers.

Content Background

What is the real number system?

A real number is a value that represents a quantity along a continuous line. The adjective real life in this context was introduced in the 17th Century by Descartes, who distinguished between real and imaginary roots of polynomials.

The number system consist of Natural Numbers, Integers, Rationals, Irrationals

1. The set of natural numbers including zero are called whole numbers - 0, 1, 2, 3, 4, 5, 6…
2. The integers are expanded set of numbers including the negatives - ….-4.-3,-2,-1, 0, 1, 2, 3, 4, 5,….
3. Rational Numbers are all numbers of the form a/b where a and b are intergers, $b \neq 0$ (b cannot be a zero)- also called fractions where a is the numerator and it's an expression for how many of e.g. fourths, fifths… etc… and b is the denominator which expresses the size of the fraction as in e.g. fourths and fifths… etc…
   The denominator cannot be a zero (but numerator can).

   The fractions like numbers smaller than 1, such as $\frac{1}{2}$, $\frac{3}{4}$ (called proper fractions) or they can be called numbers bigger than 1 (called improper fractions). And then, those that can have the denominator as 1 e.g. $3=\frac{3}{1}$

This means that all the previous sets of numbers (natural numbers, whole numbers, and integers) are subsets of the rational numbers.

Irrational Numbers

The numbers that cannot be expressed as fractions are called irrational numbers

- Cannot be expressed as a ratio of integers.
- As decimals they never repeat or terminate (rational always do one or the other)

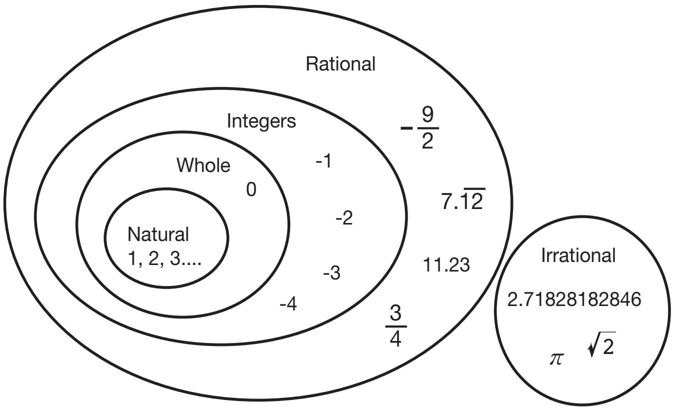
Examples

3/4=0.75 Rational (terminates)
2/3=0.666666 Rational (terminates)
5/11=0.454545(45) Rational (terminates)
5/7=0.714285(714285) Rational (terminates)
$\sqrt{2}$=1.41421356… Irrational (never repeats or terminates)
π=3.14159265… Irrational (never repeats or terminates)

Therefore Real Numbers are;

- Rational + Irrationals
- All points on the number line
- Or all possible distances on the number line

When we put the irrational numbers together with the rational numbers, we finally have the complete set of real numbers. Any number that represents an amount of something, such as a weight, a volume, or the distance between two points, will always be a real number. The following diagram illustrates the relationships of the sets that make up the real numbers.



Example 1:

Show that the following are rational numbers;

a. $2\frac{3}{4}$ b. 0.637 c. 3 d. 0.4 e. -3.1

Solutions

a. $2\frac{3}{4} = \frac{11}{4}$
This is the form $\frac{a}{b}$, where a and b both are integers, hence $2\frac{3}{4}$ is a rational number
b. $0.637 = \frac{637}{1000}$,
This is the form $\frac{a}{b}$, where a and b both are integers, hence 0.637 is a rational number
c. $3 = \frac{3}{1}$,
This is the form $\frac{a}{b}$, where a and b both are integers, hence 3 is a rational number
d. $0.4 = \frac{4}{10}$,
This is the form $\frac{a}{b}$, where a and b both are intergers, hence 0.4 is a rational number
e. $-3.1 = -3\frac{1}{10} = \frac{31}{10}$,
This is the form $\frac{a}{b}$, where a and b both are integers, hence -3.1 is a rational number

Example 2.
Determine whether the following real numbers are rational or irrational

a. $\sqrt{6}$ b. $\sqrt{\frac{16}{49}}$

Solution

a. $\sqrt{6} = 2.44989743\ldots$

Since the decimal neither terminates nor recurs, it cannot be expressed as the ratio of two integers, hence $\sqrt{6}$ is an irrational number

b. $\sqrt{\frac{16}{49}} = \frac{16}{49}$ (since $\frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$)

This is the form $\frac{a}{b}$, where a and b both are intergers, hence $\sqrt{\frac{16}{49}}$ is a rational number

The properties of Surds
Irrational numbers that contain the radical sign √ are called surds. When working with surds we may use the following properties: If $x>0$ and $y>0$,

$(\sqrt{x})^2 = x = \sqrt{x^2}$
$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Example:

Simplify

a. $(\sqrt{5})^2$ b. $\sqrt{5^2}$ c. $(3\sqrt{5})^2$

Solution

a. $\sqrt{5}$ is the positive number which multiplied by itself is equal to 5.
Hence $(\sqrt{5})^2 = 5$

b. $\sqrt{5^2} = \sqrt{25} = 5$

c. $(3\sqrt{5})^2 = 3\sqrt{5} \times 3\sqrt{5}$
$= 9 \times (\sqrt{5})^2$
$= 9 \times 5$
$= 45$

Fractional Indices

a. Use the index laws to simplify $\{a^{\frac{1}{4}}\}^4$ b. simplify $(\sqrt[4]{a})^4$
c. Hence show that $a^{\frac{1}{4}} = \sqrt[4]{a}$
a. $\{a^{\frac{1}{4}}\}^4 = a^{\frac{1\times4}{4}} = a^1 = a$ b. $(\sqrt[4]{a})^4 = (\sqrt[4]{a}) \times (\sqrt[4]{a}) \times (\sqrt[4]{a}) \times (\sqrt[4]{a}) = a$
c. Since $\{a^{\frac{1}{4}}\}^4 = (\sqrt[4]{a})^4 = a$, then $a^{\frac{1}{4}} = \sqrt[4]{a}$

Write in surd form: a. $k^{\frac{1}{5}}$ b. $z^{\frac{1}{10}}$
a. $k^{\frac{1}{5}} = \sqrt[5]{k}$ b. $z^{\frac{1}{10}} = \sqrt[10]{z}$

Index Notation

This is a short way of writing the repeated product of numbers, for example 6 × 6 × 6 × 6 × 6 may be written 6^5.This is read as ‘6 to the power 5’ or ‘6 to the fifth (power)’.
The 6 is called the base. The 5 is called the power, index or exponent. It tells us how many times the base has been repeatedly multiplied.

Scientific Notation

The distance of Mars from the sun is approximately 229 000 000 kilometres. The diameter of the hydrogen atom is 0.000 000 000 025 4 meters. Scientists invented a more convenient method of writing very large and very small numbers like the ones above. It is called scientific notation or standard notation.

To write a number in scientific notation, it is written as the product of a number between 1 and 10 and a power of 10.

Example: a. 5 000 000 = 5 × 1 000 000 = 5×10^6
b. 5300 = 5.3 x 1000 = 5.3×10^3

Unit: Indices	Topic: Operations with Rational Numbers
Benchmark	**9.1.1.10** Apply addition, subtraction, multiplication, division and order of Operations when calculating with rational numbers.

Learning Objectives: By the end of the topic, students will be able to;
- perform numeric computations with rational numbers, and
- solve problems with rational numbers using the order of operations.

Essential questions:
- What are the rules of adding, subtracting, multiplying and dividing rational numbers?

Key Concepts (ASK-MT)	
Attitudes/Values	• Show confidence in applying mathematical knowledge in performing numerical computations.
Skills	• Perform numerical computations with rational numbers.
Knowledge	• Properties of Rational Numbers. • Order of Operations.
Mathematical Thinking	• Think about how to identify and express the main ideas of operations with rational numbers.

Content Background

1. Addition and Subtraction

i. Convert all mixed numbers to improper fractions.
ii. Find the LCM of their denominators.
iii. Express the fractions as equivalent fractions using the LCM as the new denominator.
iv. Then add or subtract the numerators.

Example: (a) $1\frac{1}{6} + \frac{1}{3} = \frac{7}{6} + \frac{1}{3} = \frac{7+2}{6} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$

(b) $2\frac{3}{4} - 1\frac{1}{3} = \frac{11}{4} - \frac{4}{3} = \frac{33-16}{12} = \frac{17}{12} = 1\frac{5}{12}$

2. Multiplication

i. Convert all mixed numbers to improper fractions.
ii. Cross out the common factors whenever possible.
iii. Multiply the numerators first and then the denominators.
iv. Give the answer in the simplest form.

Example: $2\frac{2}{15} \times 3\frac{3}{8} = \frac{32^4}{15^5} \times \frac{27^9}{8^1} = \frac{4 \times 9}{5 \times 1} = \frac{36}{5} = 7\frac{1}{5}$

3. Division

i. Convert all mixed numbers to improper fractions.
ii. Invert the second fraction and then multiply with the first one.

Example: $\frac{2}{3} \div 6 = \frac{2}{3} \div \frac{6}{1} = \frac{2^1}{3} \times \frac{1}{6^3} = \frac{1}{9}$

Unit: Indices	Topic: Laws of Indices
Benchmark	**9.1.1.11** Apply the index laws to variables, using positive integer indices and the zero.

Learning Objectives: By the end of the topic, students will be able to;

- perform numerical computations using the laws of indices,
- solve related problems using the index laws.

Essential questions:

- What are the laws of Indices?
- How can we apply the index laws?

Key Concepts (ASK-MT)	
Attitudes/Values	• Show confidence in applying mathematical knowledge in performing numerical computations. • Think independently in solving various types of problems.
Skills	• Perform numerical computations with the index laws. • Solve problems using the index laws.
Knowledge	• Indices, Zero Index, Negative Indices, Law of Indices.
Mathematical Thinking	• Think about how to identify and express the main ideas of operations with Indices summarize key points about the index laws.

Content Background

Laws of Indices

Law 1: Multiplying Indices

Consider $3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3)$

$= (3 \times 3 \times 3 \times 3 \times 3 \times 3)$

$= 3^6$ (which is the same as 3^{2+4})

Therefore, when multiplying powers of the same base, add the indices ($a^m \times a^n = a^{m+n}$)

Law 2: Dividing Indices

Consider $\frac{2^5}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$

$= 2 \times 2$

$= 2^2$ (which is the same as 2^{5-3})

To divide powers of the same base, subtract the indices ($a^m \div a^m = a^{m-n}$).

Law 3: Raising a power to a power

Consider $(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

$= (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3)$

$= 3^8$ (which is the same as $3^{2\times4}$)

When raising the power of a quantity to a power, multiply the indices $(a^m)^n = a^{mn}$

Law 4: Distribution of Powers

Consider $(2 \times 3)^4 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2^4 \times 3^4$

In general $(a \times b)^m = a^m \times b^m$ and $(a \div b)^m = (\frac{a}{b})^m = \frac{a^m}{a^m}$

Examples

a. $2^4 \times 3^4 = (2 \times 3)^4$
$= 6^4$
$= 1296$

b. $a^3 \times b^3 = (a \times b)^3 = (ab)^3$

c. $(\frac{3}{4})^2 = \frac{32}{42} = \frac{9}{16}$

d. $(\frac{3a^4}{c^2})^3 = \frac{3^3 a^{4x3}}{c^{2x3}}$
$= \frac{27a^{12}}{c^6}$

A negative index indicates the reciprocal of the quantity ($a^{-m} = \frac{1}{a^m}$)

Example: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Any number raised to the power of zero is 1.

Example: a. $(23y)^0 = 1$ b. $23^0 = 1$

Strand 2: Geometry, Measurement and Transformation

Content Standard: Students will be able to comprehend the meaning and significant of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyze mathematical situations

Units	Benchmark	Topics	Lesson Titles
Mensuration of Plane figures and Solids	**9.2.2.1** Apply metric systems of measurements to solve problems.	Metric System of Measurement	• Metric systems of measurement • Estimating Length
	9.2.2.2 Calculate the perimeter and areas of plane figures and composite shapes.	Perimeter	• Perimeter of Plane Figures • Circumference of a Circle and Sector • Perimeter of composite shapes
	9.2.2.3 Use formulas, including appropriate units of measure, to determine the surface area and volume of selected prisms, cylinders, cones, and pyramids.	Area of Plane Figures	• Area of Triangles and Quadrilaterals • Area of Circles and Sectors • Area of Composite Shapes
	9.2.2.4 Solve problems involving the surface area and volume of right prisms and composite solids.	Surface Area and Volume	• Nets and Surface Area of Prisms • Nets and Surface Area of Cylinders • Nets and Surface Area of Pyramids • Volume of Prisms • Volume of Cylinders • Volume of Pyramids
	9.2.2.5 Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	Surface Area and Volume of Right Solids	Surface area and volume of; • right pyramids, • right cones, • spheres and • related composite solids
Rates	**9.2.2.6** Investigate very small and very large time scales and intervals.	Time Scales	• Time and Rates • Time scales and intervals
	9.2.2.7 Explain representation of tables and graphs of rates.	Rate, Tables and Graphs	• Measure of Rates • Rates, tables and graphs
	9.2.2.8 Solve problems with rates and interpret related graphs.	Travel Graphs	• Distance – Time Graphs • Speed- Time Graphs • Practical applications of rates of change

Geometry	**9.2.2.9** Explain the concepts of parallel lines, perpendicular lines, angles, and angles sum of polygons.	Angles and Lines	• Angles formed by Parallel lines and a transversal • Perpendicular lines • Angle sum of polygons
	9.2.2.10 Use the enlargement transformation and the conditions for triangles to explain similarities and solve problems.	Transformations including reflections, rotation and translation	• Types of Transformation of shapes • Reflection • Rotation • Translation
	9.2.2.11 Perform transformations including reflections, rotation and translation and describe the size, position and orientation of the resulting shapes.	Congruency and similarity	• Measures of congruency • Properties of similarity • Congruency and similarity of plane figures
	9.2.2.12 Investigate angles formed by chords and tangent of a circle.	Circles	• Parts of a circle • Angles formed by chords • Angles formed by Tangent

Unit: Mensuration of Plane Figures and Solids	Topic: Metric System of Measurement
Benchmark	**9.2.2.1** Apply metric systems of measurements to solve problems.

Learning Objective: By the end of the topic, students will be able to;

- solve problems using metric systems of measurement of length, mass, area, volume and capacity.

Essential questions:
- What is needed to determine that solve problems in length, mass, area, volume and capacity?

Key Concepts (ASK-MT)	
Attitudes/Values	• Participate critically and appreciate metric system of measurements.
Skills	• Review the systems of measurement. • Evaluate and convert systems of measurement.
Knowledge	• Systems of measurement in length, mass, area, volume, capacity and conversion units.
Mathematical Thinking	• Think about how to solve problems using metric systems of measurement through reflection, reasoning, problem solving and assimilating.

Content Background

1. Conversion

Length	Mass
10 millimetres (mm) = 1 centimetre (cm) 1 000 millimetres (mm) = 1 metre (m) 100 centimetres (cm) = 1 metre (m) 1 000 metres (m) = 1 kilometre (km)	1 000 mg (mg) = 1 gram (g) 1 000 grams (g) = 1 kilogram (kg) 1 000 kilograms (kg) = 1 tonne (t)

Note: Your child may be confused by the idea of mass and how it is different from weight. Explain that mass is never affected by anything like gravity. Your weight may change when you are under water in a pool but your mass remains the same!

2: Converting Metric Mass Units

Milligram (mg) is the smallest commonly used mass unit and kilograms (kg) is the largest.
1,000 mg = 1 g; 1,000 g = 1 kg

When going from a small unit to a large unit, you divide:
Convert 5,000 milligrams into grams.
5,000 ÷ 1,000 = 5 g

When going from a large unit to a small unit, you multiply:
Convert 7 kilograms into grams?
7 x 1000 g = 7000 g

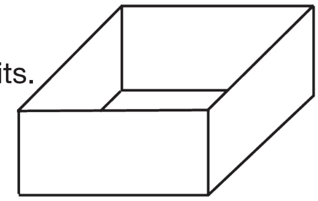
3: Volume and Capacity

Conversion factors for volume
1 000 millilitres (ml) = 1 litre (ℓℓ)
1 000 litres (ℓℓ) = 1 kilolitre (kl)

Capacity is the amount of liquid a container can hold. In other words, capacity is the volume of a container given in terms of liquid measurement.

Examples of Capacity

The figure shows the tub in which Susan usually stores water for the cows. The capacity of the tub could be about 5 liters.



Find the capacity of a cylinder with base radius of 5 units and height 7 units.
Step 1: base radius = 5 and height = 7
Step 2: Capacity of cylinder = volume = π x 52 x 7 = 549.8 cubic units

4: Solving problems

Distance Word Problem
Jessica is measuring two line segments. The first line segment is {30 cm long}. The second line segment is {500 mm long}. How long are the two line segments together? (Answer in cm)

Step 1: Underline key words and {bracket} important numbers.
Step 2: Ask: What is this question asking me to do? (Find the length of both line segments together).
Step 3: Since the line segments are in different units, convert mm into cm. 500 mm = 50 cm because there are 10 mm in 1 cm, 20 mm in 2 cm, 30 mm in 3 cm, 40 mm in 4 cm etc.
Step 4: Add to solve since together is an addition word (50 cm + 30 cm = 80 cm)

Metric Mass Units Word Problem

1. Ezra's stuffed animal has a mass of {300 grams}. How many milligrams is the stuffed animal?

Step 1: Underline the important information and bracket the numbers.
Step 2: Decide whether you convert the units by multiplying or dividing.
Step 3: Remember that grams are larger than milligrams so you multiply. 300 x 1,000 = 300,000 mg

2. Skylar goes to a pumpkin patch and picks out a pumpkin that has a mass {6,000 grams}. How many kilograms is the pumpkin?

Step 1: Underline the important information and bracket the numbers.
Step 2: Decide whether you convert the units by multiplying or dividing.
Step 3: Remember that grams are smaller than kilograms so you divide. 6,000 ÷ 1,000 = 6 kg

3. A drum contains 12 litres 156 ml of varnish. It is supplied to 12 shops equally. How much varnish does each shop get?

Solution:
Varnish distributed to 12 shops = 12 l 156 ml
Varnish given to 1 shop = (12 l 156 ml) ÷ 12
= 12156 ml ÷ 12
= 1013 ml
= 1113 ml
Each shop will get varnish = 1113 ml

Unit: Mensuration of Plane Figures and Solids **Topic: Perimeter**

Benchmark	**9.2.2.2** Calculate the perimeter and areas of plane figures and composite shapes.

Learning Objective: By the end of the topic, students will be able to;

- evaluate and calculate the perimeter of plane figures, circle and composite shapes.

Essential questions:

- What factors are needed to determine the perimeter?
- How do you calculate the perimeter of plane figures, circle and composite shapes?

Key Concepts (ASK-MT)	
Attitudes/Values	• Rationally and confidently calculate perimeter.
Skills	• Calculate perimeters of plane figures, circumference, sector
Knowledge	• Unit of length and Perimeter. • Plane figures, Circle, circumference, sector, composite shapes.
Mathematical Thinking	• Think about how to calculate the perimeter of plane figures, circles and composite shapes through reasoning and problem solving.

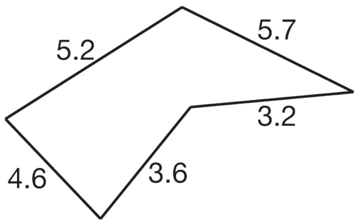
Content Background

Perimeter

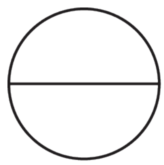
The distance around the closed shape is called the perimeter. It is usually measured in millimeters, centimeters or meters. The perimeter of a circle is called the circumference and the formula is $C=\pi d$ or $C=2\pi r$

Example:

Find the perimeter of these shapes (all measurements in centimeters)



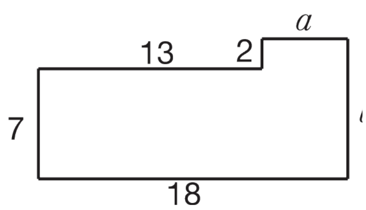
P= 5.7+5.2+4.6+3.2+3.6 = 22.3 cm



Circle with diameter of 12 cm

$C = \pi d$

π x 12 = 37.7 cm



Find side a

a = 18-13

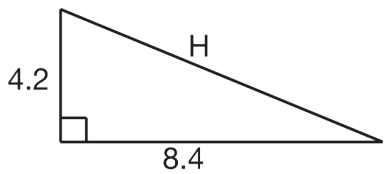
a = 5,

Find side b

b = 7 +2

b = 9

P= 5.7+5.2+4.6+3.2+3.6 = 22.3 cm



Find side H

$H^2 = A^2 + B^2$

$H^2 = 8.4^2 + 4.2^2$

$= 88.42$

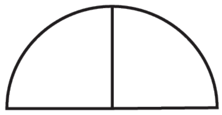
$H = \sqrt{88.42}$

$= 9.4$ cm

P = 8.4 + 4.2 + 9.4 = 22 cm

Perimeter of Composite figures

a.



If Diameter is 6 cm, find the perimeter of the above semi-circle

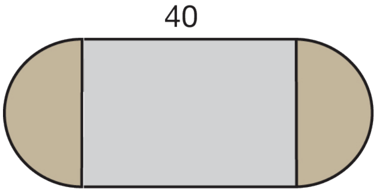
$= (2\pi r \div 2) + 6)$

$= \pi \times 3 + 6$

$= 3\pi + 6$

$= 15.4$ cm (1.d.p)

b.



Circumference of a circle + 2 (length of a straight line)

$= \pi \times 40 + 2 \times 40$

$= 40\pi + 80$

$= 205.7$ m

Problems of Perimeters

A farmer decides to fence a 400 m by 350 m paddock with a 4-strand wire fence. Find the total cost of the wire required given that single strand wire costs 12.4 toea per meter

Find the perimeter of the paddock;

P = 2 (400 + 350)

= 1500 m

Length of wire to be bought = 1500 x 4 = 6000 m

Then find the cost of the wire 6000 x 12.4 = 74,400

Convert to Kina 74,400 ÷ 100 = K744.00

Find the total length of string used to tie a box with base of 15cm, length of 20cm and height of 10cm. An extra 15cm is required to tie the knot and bow

Perimeter of box = 2 (15 +10) + 2 (20 +10)

= 50 cm + 60 cm = 110 cm

The length of string needed = 110 cm +15 cm = 125 cm

Unit: Mensuration of Plane Figures and Solids	Topic: Area of Plane Figures
Benchmark	**9.2.2.3** Use formulas, including appropriate units of measure, to determine the surface area and volume of selected prisms, cylinders, cones, and pyramids.

Learning Objective: By the end of the topic, students will be able to;

- calculate the Area of triangles, quadrilaterals, circles and composite shapes.

Essential questions:

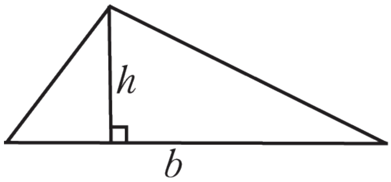
- What is needed to calculate the perimeter of plane figures and composite shapes?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate formulae relating to plane figures and composite shapes.
Skills	• Calculate the Area of Triangles, Quadrilaterals, circles, sector and composite shapes.
Knowledge	• Area, Triangles, quadrilaterals. • Area, circles, center of circle, radius, diameter, sector. • Area, composite shapes.
Mathematical Thinking	• Think about how to calculate the area of triangles, quadrilaterals, circles and composite shapes through problem solving and reflection.

Content Background

Area of special quadrilaterals

1. Area of Triangle = $\frac{1}{2}$ × base × height



2. Rhombus

Area = $\frac{1}{2}$ x product of the lengths of the diagonals

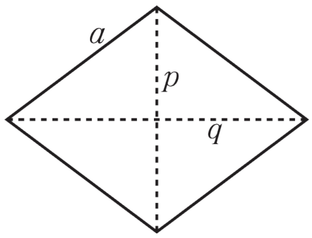
A = $\frac{1}{2}$ x xy, where x and y are the lengths of the diagonals (p and q)

Example: Find the area of rhombus with the diagonals having lengths of 5 cm and 7 cm

$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 5 \times 7\ cm^2$

$= 17.5\ cm^2$



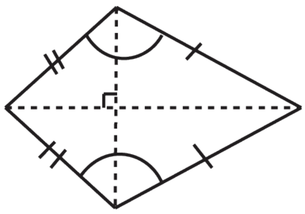
3. Kite

Area = $\frac{1}{2}xy$

Find the area of the Kite 5cm and 8 cm

$A = \frac{1}{2} \times 5 \times 8\ cm^2$

$= 20\ cm^2$



4. Trapezium

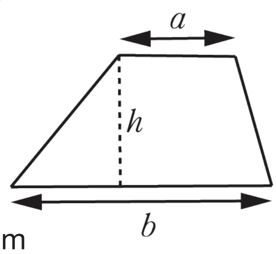
$A = \frac{1}{2} \times ah + \frac{1}{2}bh$

$= \frac{1}{2}h(a+b)$

$= (\frac{a+b}{2})h$

Note: First find the height then use the formula

Example: find the area of a trapezium with a = 11 m, b = 16 m, h = 4 m



$A = (\frac{11+16}{2})\ m \times 4\ m = 13.5 \times 4 = 54\ m^2$

Composite Areas

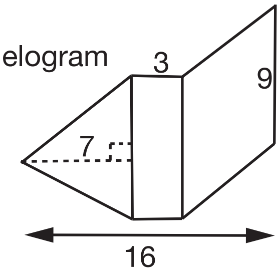
Figures that cannot have their areas calculated using one formula are called composite areas
The area of a composite figure can be calculated by dividing it into identifiable shapes, then adding or subtracting the area of these shapes to find the total area.
Find the area of this shape;

The area is found by adding the area of the triangle, the rectangle and parallelogram
Total area = area of triangle + rectangle + parallelogram

$= (\frac{1}{2} \times 7 \times 9) + (3 \times 9) + (\frac{1}{2} \times 9 \times 6)$

$= 31.5 + 27 + 27$

$= 85.5 \text{ cm}^2$



Find the areas of shaded areas
The shaded area is found by calculating the total area and then subtracting the unshaded area

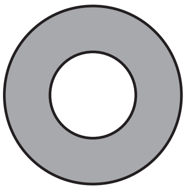
Area = area of the large area – area of the small area

$A = \pi r^2 = \pi \times 5 \times 5 - \pi \times 3 \times 3$

$= 25\pi - 9\pi$

$= 16\pi = 16 \times 3.14$

$= 50.24 \text{ cm}^2$



Problems on area of rectangles

The length of rectangular plot is $5\frac{1}{3}$ times that of its breadth. If the area of the plot is 270 square meters, then what is its length?

Let breadth be x meters, then length be $\frac{16x}{3}$

$x \times \frac{16x}{3} = 270$

$x^2 = \frac{1800}{16}$

$x = \frac{90}{4}$, then length= $\frac{16}{3} \times \frac{90}{4} = 120$ m

The ratio of the length and breadth of a plot is 4:3. If the breadth is 40 m less than the length. What is the perimeter of the plot?

Let x be 4x meters, then breadth be 3x meters
Then 4x - 3x = 40 => x = 40
Length l = (4 x 40) = 160 m, breadth b = (3 x 40) = 120 m
Perimeter = 2(160 +120) = 2(280)= 560 m

A rectangular carpet has an area of 120 square meters and a perimeter of 46meters. The length of its diagonal is;

$l \times b = 120 \text{ m}^2$ and 2 (l x b) = 46 = > (l x b) = 23
$(l - b)^2 = (l + b)^2 - 4lb - (23)^2 - 4 \times 120 = (529 - 480) = 49 \Rightarrow l - b = 7$
On solving;
l + b = 23, l - b = 7, we get l = 15, b = 8
Diagonal is $\sqrt{((15)^2)} + 8^2$

$= \sqrt{(225+64)} = \sqrt{289} = 79$ m

Unit: Mensuration of Plane Figures and Solids	Topic: Surface Area and Volume
Benchmark	**9.2.2.4** Solve problems involving the surface area and volume of right prisms and composite solids.

Learning Objectives: By the end of the topic, students will be able to;

- apply formula and appropriate units to determine the Net and surface area of prisms, cylinders and pyramids, and
- apply formula and appropriate units to determine the Volume and capacity of prisms, cylinders, cones and pyramids.

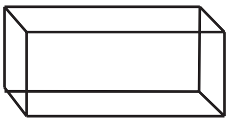
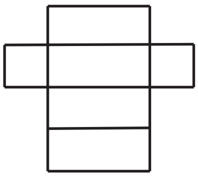
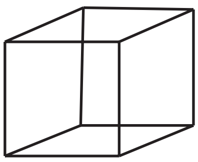
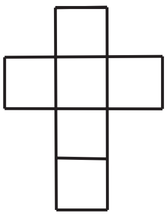
Essential questions:

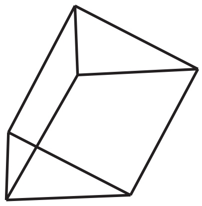
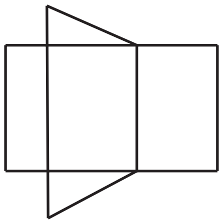
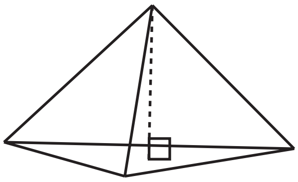
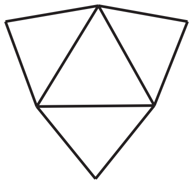
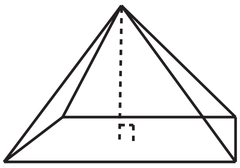
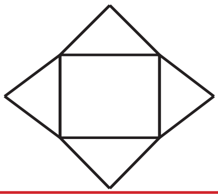
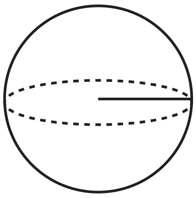
- What skills and processes needed to calculate the area of triangle, quadrilateral, circles and sector?
- What are the parts of a circle that will be used to interpret a circle?
- What other tools will be used to solve with?

Key Concepts (ASK-MT)	
Attitudes/Values	• Relate and Appreciate nets. • Appreciate and confidently use formula to calculate surface area and volume.
Skills	• Analyse and evaluate the nets and calculate the surface area using formulae. • Calculate the volume and capacity of selected prisms, cylinders, cones and pyramids using formulae.
Knowledge	• Formulae of surface area and volumes of nets and various figures and their calculations.
Mathematical Thinking	• Think about how to apply formula to calculate surface area and volume of prisms and composite solids through logical reasoning, problem solving and reflection.

Content Background

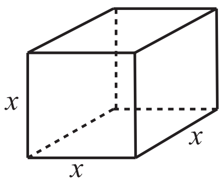
1. 3 Dimensional Shapes with Corresponding Ntes

Shape	Net	Formula
Right Rectangular Prism 		Surface Area (SA) $= 2(wl + hl + hw)$ w = width l = length h = height Volume (V)= Lwh
Cube 		SA = $6a^2$ a = edge length $V = a^3$

Right Triangular Prism 		SA=$wh + lw + lh + ls$) w = width h = height l = length s = side
Right Triangular Pyramid 		SA = $4(\frac{1}{2}\ bh)$
Square Pyramid 		SA = $A_{base} + 4(A_{side})$ A_base = b^2 A_side = $\frac{1}{2}$ bh b = base h = height
Sphere 		$V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

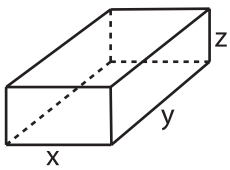
2. Surface Area and Volume of 3-D Shapes

Cube



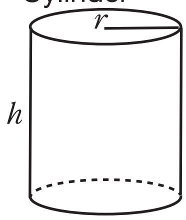
$V = x^3$
$SA = 6x^2$

Cuboid



V = xyz
SA = $2y + 2z + 2yz$

Cylinder

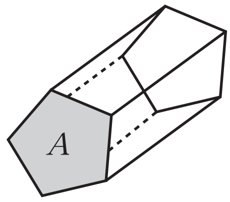


$V=\pi r^2 h$

Area of curved surface
$=2\pi rh$
Area of each end
$= \pi r^2$

Total surface area
$=2\pi rh+2\pi h$

Prism



A prism has a uniform cross-section

Volume = area of cross section × length = Al

Example 1

Calculate the surface area of the cuboid shown.

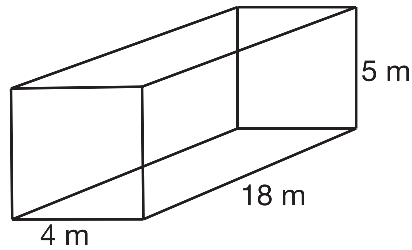
$A = 2(4 \times 5)m^2 + 4(18 \times 5)m^2$

$A = 40\ m^2 + 360\ m^2$

$A = 400\ m^2$

Calculate the volume of the cuboid shown.

$V = (4 \times 18 \times 5)\ m^3 = 360\ m^3$



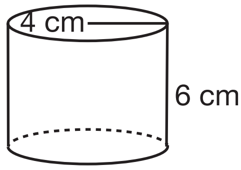
Example 2

Calculate the volume and total surface area of the cylinder shown.

$V = \pi r^2 \times 6$

$= 3.14 \times 4 \times 4 \times 6$

$= 301.44\ cm^3$



$A = 2\ (3.14 \times 16)\ cm^2 + (6 \times 8)\ cm^2$

$A = 2 \text{ X } 50.24 + 48\ cm^2$

$A = 148.48\ cm^2$

Example 3

Calculate the volume and total surface area of the triangular prism given below

$V = \frac{1}{2}\ (6 \times 8) \times 10$

$= 24 \times 10$

$= 240\ cm^2$

$A = 2 \times \frac{1}{2}\ (6 \times 8) + 3\ (10 \times 6)$

$= 2 \times 24 + 3 \times 60$

$= 228\ cm^2$

Unit: Mensuration of Plane Figures and Solids	Topic: Surface Area and Volume of Right Solids
Benchmark	**9.2.2.5** Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Learning Objectives: By the end of the topic, students will be able to;

- apply formula and appropriate units to determine the surface area and volume of right pyramids, cones, spheres and related composite figures.

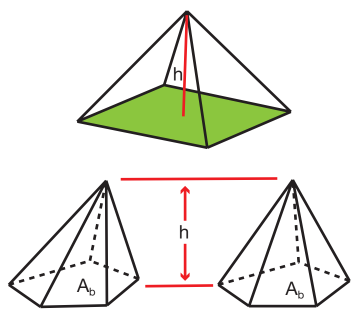
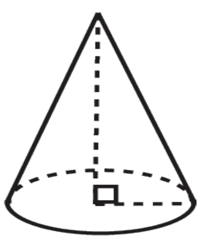
Essential questions:

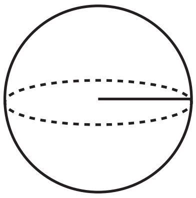
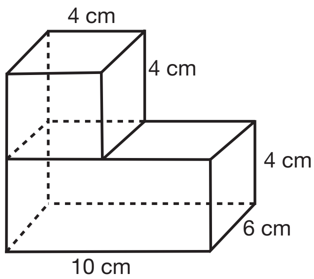
- What skills and processes are needed to calculate the surface area of right pyramids, cones, spheres and composite figures?
- What other tools will be used to solve and calculate?

Key Concepts (ASK-MT)	
Attitudes/Values	• View and appreciate the figures. • Appreciate and confidently use formulae to calculate the surface area and volume.
Skills	• Analyse and evaluate the surface area and using formulae to calculate the surface area and volumes of right pyramids, right cones, spheres and related composite solids.
Knowledge	• Formulas of surface area and volumes of nets and various figures and their calculations.
Mathematical Thinking	• Think about how to apply formulae to calculate surface area and volume of pyramids, cones, spheres and composite solids through logical reasoning, problem solving and reflection.

Content Background

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids

Solid Shape	Name	Definition
 Oblique Pyramid Right Pyramid	A Right Pyramid	A right pyramid is a 3-D shape that has the apex (point of the pyramid) directly above the center of the base An oblique pyramid is a 3-D shape that has the apex that doesn't line up directly above the center of the base
	A Right Cone	Cones are similar to pyramids except that their bases are circles instead of polygons. Has a circular base and an apex right above the center of the base of the circle.

<table>
<tr><td></td><td>A Right Sphere</td><td>Spheres are solids that are perfectly round and look the same from any direction</td></tr>
<tr><td>
</td><td>Composite Figures</td><td>The volume of composite solid is the sum of the volume of its parts</td></tr>
</table>

1. To find surface area and volume of Right Pyramids;

Know the,

- formula for calculating area of 2-D shapes (triangles, squares and rectangles)
- formula for calculating the volume of pyramids
- definition of volume and surface area

Understand that;

- formulas can be used quickly to find the surface areas and volume of pyramids
- then do the calculation to find the surface areas and volume of pyramids.

Volume of right pyramids;

V = $\frac{(lwh)}{3}$, l (base length), w (base width), h (pyramid height)

2. Surface Area and Volume of Right Cones

Know the,

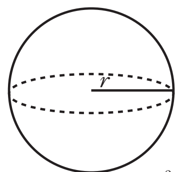
- formula for calculating area of the base of a cone (which is a circle)
- formula for calculating the volume of cone.
- definition of volume and surface area

V = $\frac{1}{3}\pi r^2$ x h ($\frac{1}{3}$ area of base x height)

Understand that;

formulas can be used quickly to find the surface areas and volume of cones
then do the calculation to find the surface areas and volume of cones.
volume of right cones;

3. Surface Area and Volume of Right Spheres



$Volume = \frac{4\pi r^3}{3}$

$Surface\ Area = 4\pi r^2$

Know the,

- formula for calculating surface area of a sphere ($4\pi r^2$)
- formula for calculating the volume of sphere
- definition of volume and surface area

  Volume = $\frac{(4\pi r^3)}{3}$

Understand that;

- formulas can be used quickly to find the surface areas and volume of spheres
- then do the calculation to find the surface areas and volume of spheres

4. Surface Area and Volume of composite figures

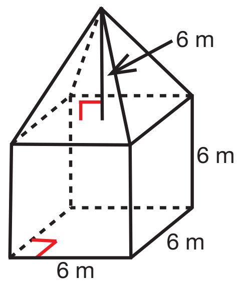
To find the volume of composite solids

- Given the figure – identify each figure in a solid
- Write the volume formulas for each figure

Add the formulas

- Substitute into the formulas
- Find the sum or answer of the volume of composite solid.

Example: Given a solid



Solution;

Volume of solid = Volume of cube + Volume of pyramid

$V = s^3 + \frac{1}{3}bh$

$V = 6^3 + \frac{1}{3} \times (6)^2 \times 6$

$V = 216 + 72$

$V = 288 \text{ m}^3$

Unit: Rate	Topic: Time Scales
Benchmark	**9.2.2.6** Investigate very small and very large time scales and intervals.

Learning Objectives: By the end of the topic, students will be able to;

- investigate and evaluate very small intervals of time and rates, and
- investigate and evaluate very large time scales and intervals of time and rates.

Essential questions:

- What are very small and very large intervals of time, time scales and rates?
- What is needed to evaluate?

Key Concepts (ASK-MT)	
Attitudes/Values	• Rationality, Participatory. • Confident in using time, rates, scale and time intervals.
Skills	• Investigating and relating time, rate, scale and time intervals.
Knowledge	• Time, Rates. • Time scales, intervals.
Mathematical Thinking	• Think about and investigate time scales and intervals and evaluate through reasoning and problem solving.

Content Background

Rates
One of the important features of a ratio is that it is used to compare quantities measured in the same units. A rate compares quantities that are measured in different units

Examples:

1. Fuel consumption – car dealer will often talk about 'improved fuel economy' when trying to sell a new model. Fuel consumption or fuel economy is a measure of the distance a car travel on a specific quantity of fuel. Petrol consumption is usually measured in km/L.
   Cars are not only the things we expand fuel on. We use fuel in our homes to cook and for heating. The Gas and Fuel Corporation of Victoria –Australia provides customers with information on their fuel consumption- measured in mega joules per day.
2. Heart rate – this is measured in beats per minute.
3. Population rates- these include birth rates, death rates, migration rates, unemployment rates.
4. Workers Rates; rate of pay per hour.

Time and Rates

Time scales; instruments to accurately measure time and rate
Examples: Very small Time scales
Digital milligram pocket scales

Examples: Very Big time scales
Geological time scales

Econ	Era	Period	Epoch	m.y.
Phanerozoic	Cenozoic	Quarternary	Holocene	
			Pleistocene	-1.5
		Neogene	Pliocene	
			Miocene	-23
		Paleogene	Oligene	
			Eocene	
			Paleocene	-65
	Mesozoic	Cretaceous		
		Jurassic		
		Triassic		-250
	Paleozoic	Periam		
		cirboilcrous: Pennsylvanian		
		cirboilcrous: Mississippian		
		Devonian		
		Silurian		
		Ordovician		
		Cambrian		-540
Precambrian	Proterozeoic			-2500
	Archean			-3800
	Hadean			-4600

The recommended time scales to used to measure rate and time accurately are;
- White dwarfs, quartz clocks, atomic clocks and pulsars.

Solving time and rates
The formula to use is distance = rate x time,
for example distance a bus travels at the rate of 40 km per hour. To measure this rate the time scale that you would use is the digital watch or clock.

Example: Heart rate or pulse rates are measured in beats per minute. Follow the instructions below to calculate your heart beat.

- Find your pulse . try the inside of your wrist or the side of your neck.
- Count the beats for a period of 20 seconds. Sit quietly whilst doing this. Record your result.

Repeat this 6 times.
- You may record your rate on the table.
- Add your 6 results altogether and then divide by 2.
- The answer will be your heart beat/min

What is a Time Interval?
Duration of a segment of time without reference to when time interval begins and ends.
Time intervals may be given in seconds or fractions.

How many seconds make a minute?
1 min = 60 secs.
240/60 = 4 mins in 240 secs

How fast? How slow?

Example:
A truck may be moving at the speed of 45 km /h.
An insect may be flying at the speed of 2.4 m/s
The most common units are; km/h and m/s

1. A bus is moving along at a constant speed. It travels 180 kilometers in 3 hours.
   a) How far does it travel in 1 hour ans; 60 km
   b) Its speed is 60 km/h
   c)

Distance(km)	0	60	120	180
Time (h)	0	1	2	3

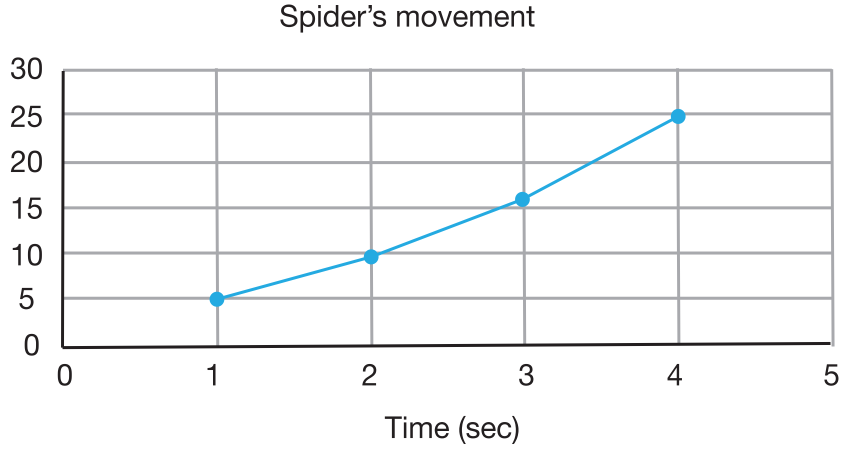
The graph of this travel rate is distance (km) over time (h)

2. A spider moves at 1cm per second. If the spider is in the corner of your classroom, find how long (in minutes) will it take to reach,

Location (points)	a	b	c	d
Speed (1cm/sec)	5	9	16	25

   a) The nearest person
   b) You
   c) The blackboard
   d) The door of the classroom

The graph of the rate of spider's movement from locations as plots will be distance (cm) over time (secs).



Unit: Rate	Topic: Rates, Tables and Graphs
Benchmark	**9.2.2.7** Explain representation of tables and graphs of rates.

Learning Objective: By the end of the topic, students will be able to;

- interpret and explain representation of tables and graphs of rates.

Essential questions:

- What are the essential skills to be used in interpreting and explaining rate tables and graphs?

Key Concepts (ASK-MT)	
Attitudes/Values	• Confident, rational.
Skills	• Evaluate and explain rates using tables and graphs.
Knowledge	• Rates, Measurement. • Rates, tables, graph.
Mathematical Thinking	• Think about and intepret tables and graphs of rates and explain with reasoning.

Content Background

Rate

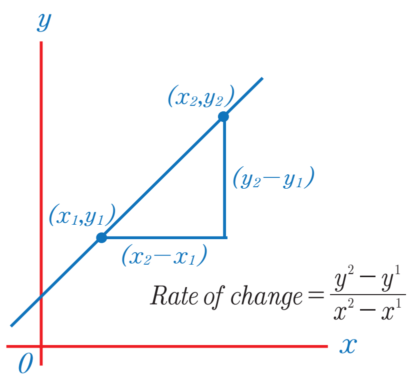
A rate is a ratio between two related quantities. Often it is a rate of change which is either negative or positive or the rate of change can be zero rate of change when the quantity does not change over time. If the unit or quantity in respect of which something is not specified, usually rate is per unit of time. However a rate of change can be specified per unit of time or per unit of length or mass or another quantity.

What is the formula of rate of change?

Average rate of change = $\frac{\textit{change in y value}}{\textit{change in x value}}$

Its general formula is:

$$[\frac{\delta y}{\delta x} = f(b) - f(a)/(b-a)]$$

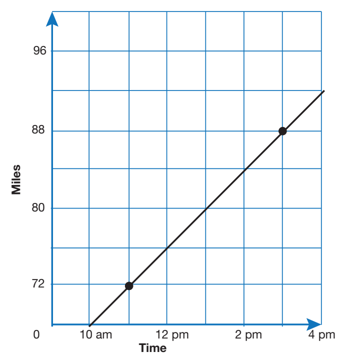


How to find the rate of change?

To find the rate of change of a line, determine the vertical change and the horizontal change. Write the rate of change as a fraction, placing the vertical change over the horizontal change.

Finally simplify the fraction if necessary

1. Find the vertical change
   Write down the points that are given or graph the line to find the two x values and two y values.
   Subtract the second y value from the first y value to find the vertical change.
2. Find the horizontal change
   Subtract the second x value too from the first x value to find the horizontal change.
3. Find the rate of change
   Divide the vertical change by the horizontal value to find the rate of change



Example 1:

if given the equation $y = 2x + 1$, graph to find the points (-2,-3) and (1,3) are two points on the line;

- to find the vertical change, 3 – (-3) = 6
- to find the horizontal change, 1- (-2) = 3
- to find the rate of change , $\frac{6}{3}$ or 2
- Check the answer by performing the method using two other points on the line.

The rate of change of 2 is the same for all points along $y = 2x + 1$

Constant Rate of Change

Example 2: Find the constant rate of change

a)

Side Length(input)	Perimeter (output)
1	4
2	8
3	12
4	16

For every increase of the side length, the rate of change is 4 for the increase of the perimeter. The rate of change is constant.

b) By 10 am a vehical had travelled 72 miles and by 2pm it travelled 88 miles
Find its rate of change

$$\frac{\Delta y}{\Delta x} = \frac{88 - 72}{14 - 10} = \frac{16}{5} = 3.2$$

Average Rate of Change

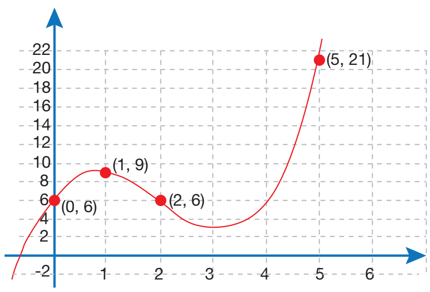
Example 3:

How to calculate average rate of change?

Use the formula:

$$\textit{Average Rate of Change} = \frac{\textit{change in Output}}{\textit{change in Input}} = \frac{f(x2) - f(x1)}{x2 - x1} = \frac{y2 - y1}{x2 - x1}$$

Use the graph to determine the average rate of change from x = 0 to x = 1 and from x = 2 to x = 5



Average Rate of Change from x = 0 to x = 1

$$\frac{9 - 6}{1 - 0} = \frac{3}{1} = 3$$

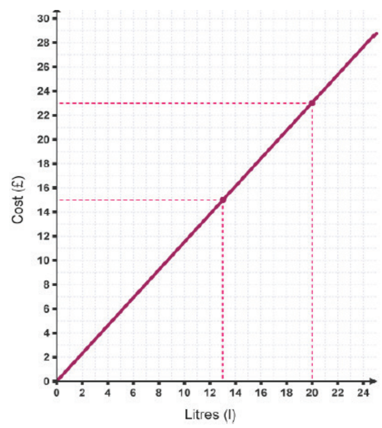
Average Rate of Change from x = 2 to x = 5

$$\frac{21 - 6}{5 - 2} = \frac{15}{3} = \frac{5}{1} = 5$$

Rate of change Problem

Example 3:

This graph shows the cost of petrol.
It shows that 20 litres will cost K23, and or K15 if you will buy 13 litres.
Using the points (0, 0) and (20, 23), the gradient = 1.15.
The units of the axes help give the gradient a meaning.
The calculation was 23–020–0= 2320 = 1.15
The gradient shows the cost per litre of Petrol is K1.15 per litre.
The graph crosses the vertical axis at (0, 0).
This is known as the intercept.
It shows that if you buy 0 liters, it will cost K0.



The slope of a line shows the rate of change in a linear relationship which is 1.15 per litre in this case. That is for every litre of Petrol that you buy will cost you K1.15, therefore if you buy 10 litres will cost you K11.50

Unit: Rate	Topic: Travels Graphs
Benchmark	**9.2.2.8** Solve problems with rates and interpret related graphs.

Learning Objective: By the end of the topic, students will be able to;

- solve problems with rates and interpret related graphs.

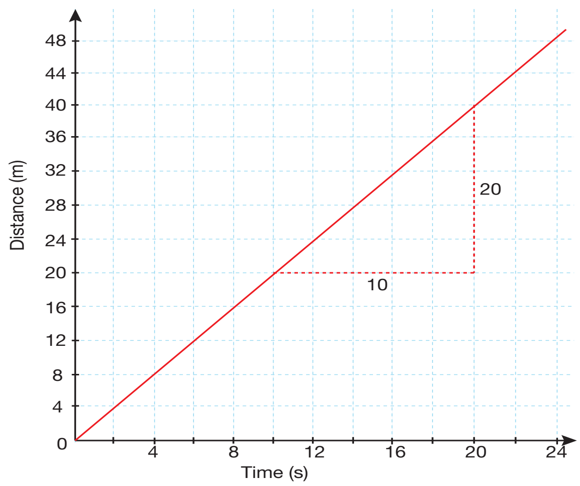
Essential questions:

- What is needed to solve problems on rates and interpret related graphs?

Key Concepts (ASK-MT)	
Attitudes/Values	• Confidently, Rationality, Participatory.
Skills	• Evaluate, interpret and solve time graph, speed-time graph and rates. • Analyze Practical problems.
Knowledge	• Distance, time, graph. • Speed, time, graph. • Rates, rates of change.
Mathematical Thinking	• Think about how to solve problems with rates and graphs.

Content Background

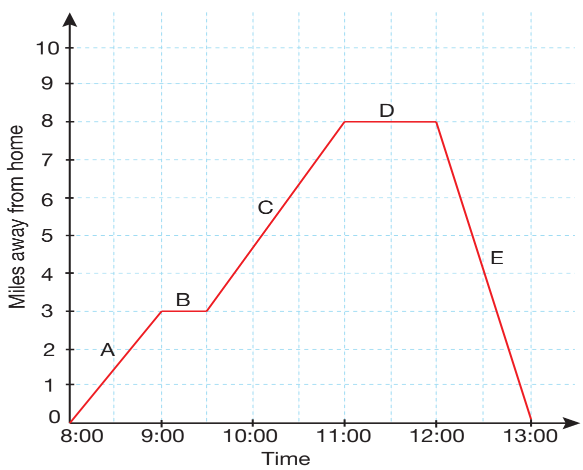
Distance-time and displacement-time graphs



Distance-time graphs show distance on the vertical axis and time on the horizontal axis. The gradient of a distance-time graph represents speed because:

$$Gradient = \frac{Rise}{Run}$$

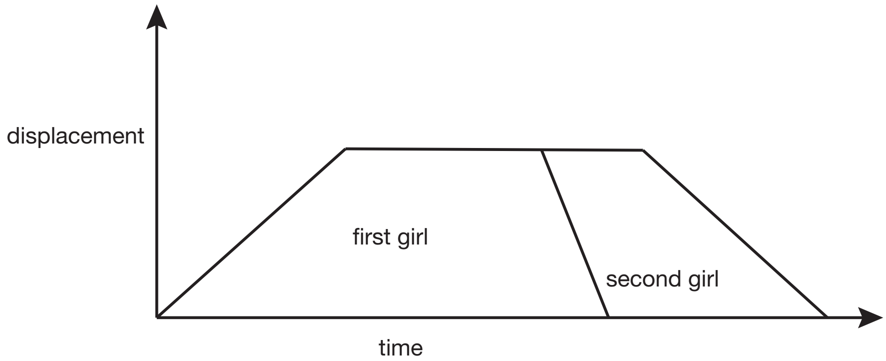
When displaying a journey, the vertical axis will often represent the distance from a particular place rather than the distance travelled. Such graphs are known as displacement-time graphs.



A. Represents travelling time from home (8.00 to 9.00am)
B. Shows a stop for at least 30 mins
C. Shows the travel time per distance within 1h 30 mins
D. Shows resting from 10.45 am to 12.00
E. Shows the travel back trip to home.

Such a story can be written to explain the graph on the left.

Two girls walk to the shop together. The first spends a few minutes at the shop and then runs home. The second remains at the shop until her friend rings her from home. She then returns walking home at a constant rate.



Unit: Geometry	Topic: Angles and Lines
Benchmark	**9.2.2.9** Explain the concepts of parallel lines, perpendicular lines, angles, and angles sum of polygons.

Learning Objectives: By the end of the topic, students will be able to;

- explain the concepts of parallel lines, perpendicular lines, angles, and
- calculate angle sum of polygons.

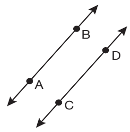
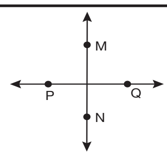
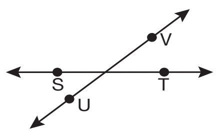
Essential questions:

- What is needed to determine that lines are parallel, perpendicular? What Angles formed by lines?
- What essentials are needed to find the angle sum of polygons?

Key Concepts (ASK-MT)	
Attitudes/Values	• Rationality, confidence.
Skills	• Investigate and define parallel lines, perpendicular lines and transversals. • Evaluate and define angle sums of polygons.
Knowledge	• Angles, Parallel lines, transversal. • Perpendicular lines. • Polygons, Angles, Angle sum.
Mathematical Thinking	• Think about how to logically explain the concepts of parallel lines, perpendicular lines, angles and angle sum of polygons.

Content Background

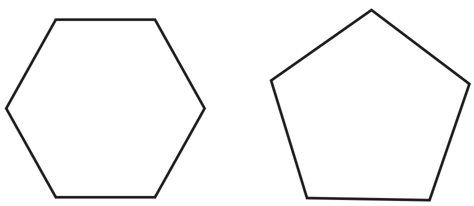
1. Parallel and Perpendicular lines

....	Description	Figure	Symbol
Parallel Lines	Two lines remain the same distance apart at all times and never intersect.		$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
Perpendicular Lines	Two lines that intersect and form right angles.		$\overleftrightarrow{PQ} \perp \overleftrightarrow{MN}$
Intersecting Lines	Intersecting lines meet or cross each other.		$\overleftrightarrow{ST}$ *Intersect* $\overleftrightarrow{UV}$

2. Polygons

A polygon is a 2D shape with at least three sides. Polygons can be regular or irregular. If the angles are all equal and all the sides are equal length it is a regular polygon.

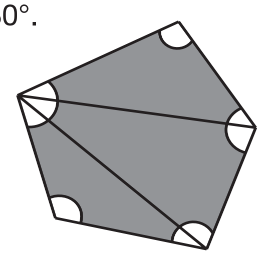
Regular Polygons

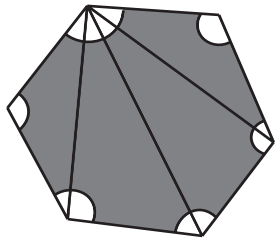


Interior angles of polygons

To find the sum of interior angles in a polygon divide the polygon into triangles.

The sum of interior angles in a triangle is 180°. To find the sum of interior angles of a polygon, multiply the number of triangles in the polygon by 180°.





Example: Calculate the sum of interior angles in a pentagon.

Solution: A pentagon contains 3 triangles. The sum of the interior angles is: 180 x 3 = 540°

The number of triangles in each polygon is two less than the number of sides. The formula for calculating the sum of interior angles is: (n-2) =180° (where n is the number of sides)

Now relate the number of sides of a polygon, the number of triangles that can be formed by drawing diagonals and the polygon's angle sum.

Polygon	Number of Vertices (n)	Number of Triangles	Angle Sum (m)
Triangle	3	1	1(180)=180
Quadrilateral	4	2	2(180)= 360
Pentagon	5	3	3(180)= 540
Hexagon	6	4	4(180)= 720
Heptagon	7	5	5(180)= 900
....	.....	.....	......
Decagon	10	8	8(180)=1440
100-gon	100	?	?
n-gon	**N**	**n-2**	**(n-2)180**

Unit: Geometry	Topic: Enlargement Transformation
Benchmark	**9.2.2.10** Use the enlargement transformation and the conditions for triangles to explain similarities and solve problems.

Learning Objectives: By the end of the topic, students will be able to;

- use the enlargement transformation and the conditions for triangles to explain similarities,
- solve problems using enlargement transformation.

Essential questions:

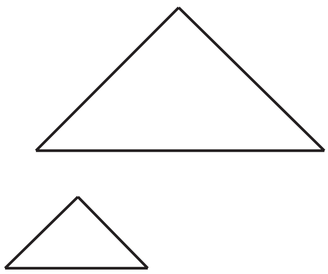
- What is needed to do enlargement transformation to solved problems?
- How do the conditions for triangles help to explains similarities?

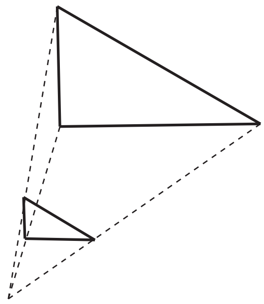
Key Concepts (ASK-MT)	
Attitudes/Values	• Critical, Rationality, confidence. • Appreciative, Rationality, confidence.
Skills	• Compare and contrast, evaluate and identify congruency and similarity in plane shapes. • Evaluate similar figures using scale factors and ratio for enlargement transformation.
Knowledge	• Triangles, congruency, similarity. • Ratios, scale factors, similar figures, Enlargement Transformation.
Mathematical Thinking	• Think about how to use enlargement transformation to solve problems.

Content Background

1. Enlargement

Enlarging a shape changes its size. All the sides of the triangle X'Y'Z' are twice as long as the sides of the original triangle XYZ. The triangle XYZ has been enlarged by a scale factor of 2.

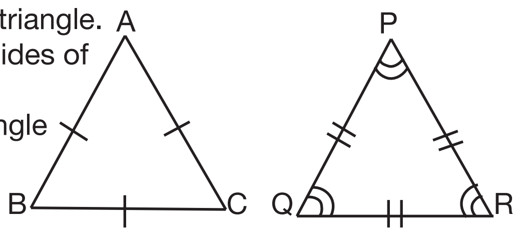




Enlargement is an example of a transformation. A transformation is a way of changing the size or position of a shape.
To enlarge a shape, a centre of enlargement is required. When a shape is enlarged from a centre of enlargement, the distances from the centre to each point are multiplied by the scale factor.
The lengths in triangle A'B'C' are three times as long as triangle ABC. The distance from O to triangle A'B'C' is three times the distance from O to ABC.

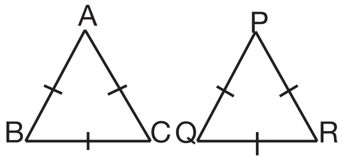
2. Congruent Triangles

Congruent triangles are similar in shape and size. The angles of one triangle will be equal to the corresponding angles of another triangle. The sides of the triangles will be equal to the corresponding sides of another triangle.
i.e. if one triangle is kept above another triangle, both the triangle will completely co-inside with each other perfectly.



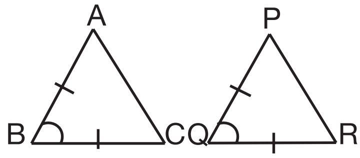
3. Properties: Congruent Triangles

SSS Congruency



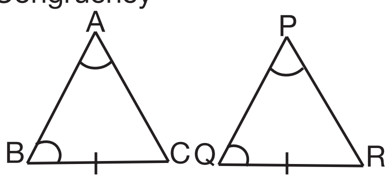
Side= Side = Side Congruency (SSS): Two triangles are congruent when three sides of a triangle are equal to the corresponding sides of the other triangle.
In the given triangle ABC and PQR,
AB = PQ, BC = QR and AC = PR.

SAS Congruency



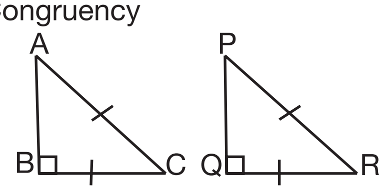
Side Angle Side Congruency (SAS):
Two triangles are congruent if two sides and an angle between them is equal to the corresponding two sides and the angle between the other triangle.
In the given triangle ABC and PQR
AB = PQ, angle B = angle Q and BC = QR

ASA Congruency



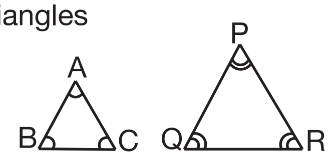
Angle Side Angle Congruency (ASA):
Two triangles are congruent if 2 angles and any one side is equal to the corresponding 2 angles and side of another triangle.
In the given triangle ABC and PQR
angle A = angle P, angle B = angle Q and BC = QR

RHS Congruency



Right Hypotenuse Side (RHS):
To check the congruency of a right angled triangle – right angle, hypotenuse and any one side of a triangle is supposed to be equal to the corresponding right angle, hypotenuse and the corresponding side of the right angled triangle.

Similar Triangles



Similar triangles are similar in shape but not in size. The angle of one similar triangle is equal to the corresponding angle of the other similar triangle. The side of one triangle is not equal to the other side of the triangle but they are in proportion

$$\frac{AB}{PQ} = \frac{BC}{GR} = \frac{AC}{PR} = \frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta PQR}$$

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = \frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta PQR}$$

The squares of the side of similar triangles are equal to the proportion of the similar triangles

Unit: Geometry	Topic: Transformations including Reflections, Rotation and Translation
Benchmark	**9.2.2.11** Perform transformations including reflections, rotation and translation and describe the size, position and orientation of the resulting shapes.

Learning Objectives: By the end of the topic, students will be able to;

- perform transformations including reflections, rotation and translation, and
- describe the size, position and orientation of the resulting shapes.

Essential questions:

- What are the types of transformations?
- What description can be drawn from the resulting shapes?

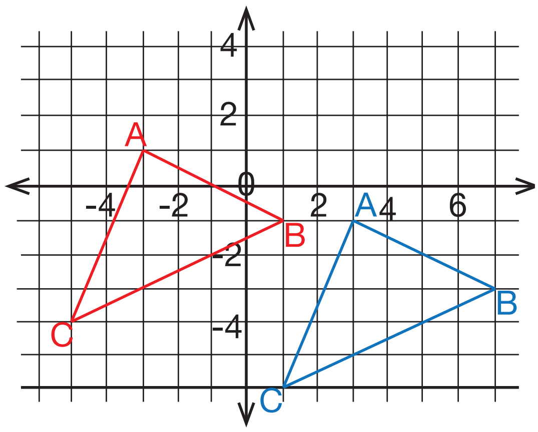
Key Concepts (ASK-MT)	
Attitudes/Values	• Confidently, Rationality, Participatory.
Skills	• Evaluate shapes using different transformations.
Knowledge	• Transformation, shapes, reflection, rotation, translation.
Mathematical Thinking	• Think about how to perform transformations using reflections, rotation and translation and reason out the resulting shapes.

Content Background

TRANSFORMATIONS

1.Translation

Translation is a geometric transformation that moves every point of a figure or a space by the same distance in a given direction.
(In Euclidean geometry a transformation is a one –to-one correspondence between two sets of points or a mapping from one plane to another.

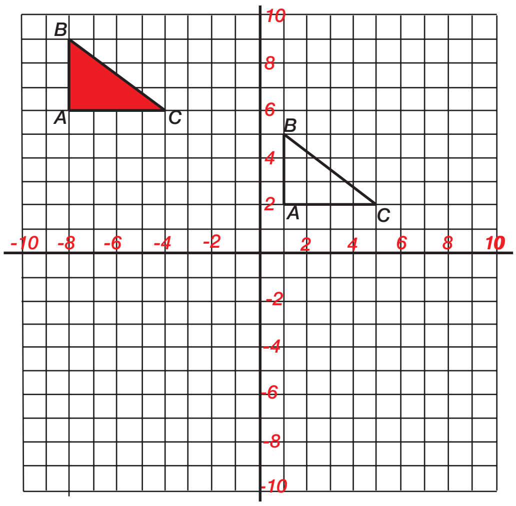


Example;

Translating a Polygon on the Coordinate Plane
Translate triangle ABC +9 units in the x-direction and -4 in the y-direction.

Method one
Write original coordinates.
A (-8, 6) x (-8 + 9), y (6 - 4)
B (-8, 9) x (-8 + 9), y (9 - 4)
C (-4, 6) x (-4 + 9), y (6 - 4)

Method two
Write the new coordinates for translated triangle 'ABC'
A' (1,2)
B' (1,5)
C' (5,2)

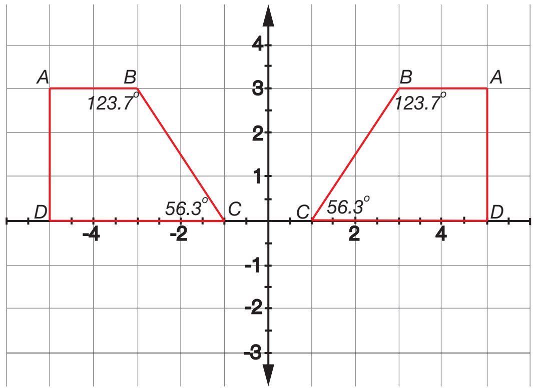


Translated shape should be congruent to the original.

2. Reflections

Few basic geometry reflection rules to help you understand and construct transformation in reflection.

- A reflection formed over a line has the potential to create a mirror image.
- All the corresponding points over the original shape are placed at the exact same distance from the mirror line or the centre.
- The size of the reflected image is the same as that of the original figure.



Example;

Reflect triangle across the y-axis

A (-7,-6)

B (-4,-6)

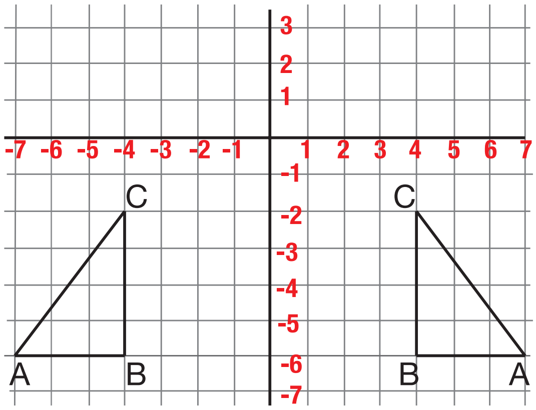
C (-4,-2)

A' (7,-6)

B' (4,-6)

C' (4,-2)

Signs of x –values will change for any reflection over the y-axis



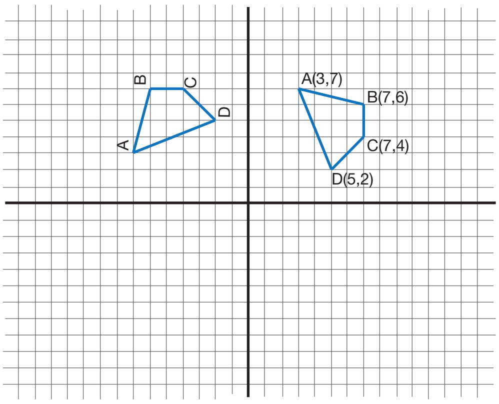
3. Rotations

Rotation of an object in two dimensions around a point 0. Rotation in mathematics is a concept originating in geometry. Any rotation is a motion of a certain space that preserves at least one point. It can describe, for example, the motion of a rigid body around a fixed point.

Type of rotation	Points on the pre-image	Points on the image (after rotation)
Rotation of 90° (clock wise)	(x,y)	(y,- x)
Rotation of 90° (counter clock wise)	(x,y)	(- y, x)
Rotation of 180° (clock wise & counter clock wise)	(x,y)	(- x,- y)
Rotation of 270° (clock wise)	(x,y)	(- y, x)
Rotation of 270° (counter clock wise)	(x,y)	(y,- x)

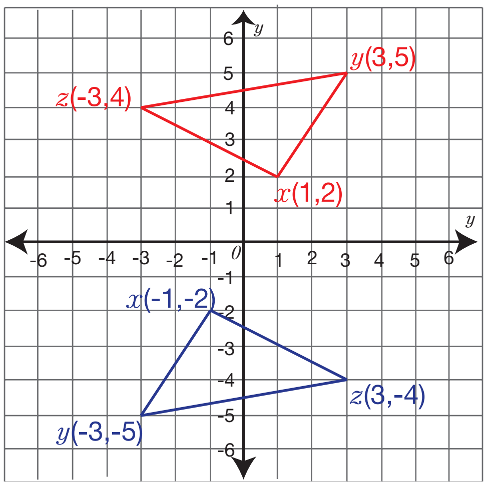
Example 1:

A rotation of a quadrilateral ABCD to 'ABCD' at 90° in the clockwise direction as shown in the diagram below;



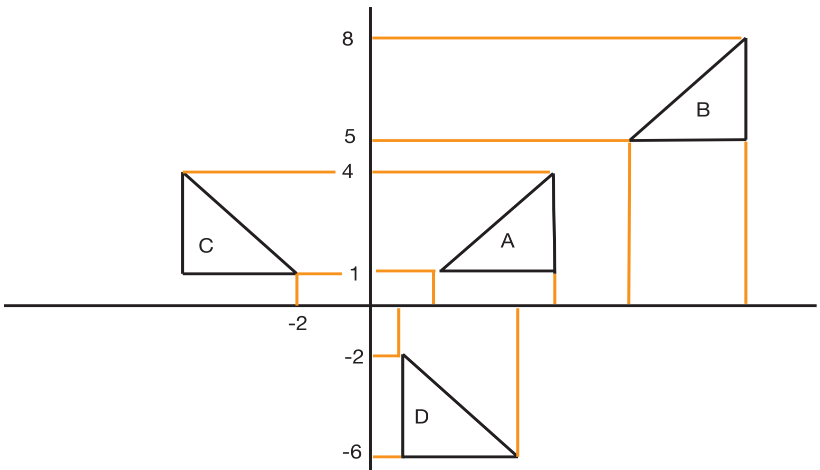
When describing a rotation, the centre and angle of rotation are given. If you wish to use tracing paper to help with rotations: draw the shape you wish to rotate onto the tracing paper and put this over shape. Push the end of your pencil down onto the tracing paper, where the centre of rotation is and turn the tracing paper through the appropriate angle (if you are not told whether the angle of rotation is clockwise or anticlockwise, it would usually be anticlockwise). The resultant position of the shape on the tracing paper is where the shape is rotated to.

Example 2:
A rotation of triangle XYZ to triangle 'XYZ' at 180° in clockwise & counter clockwise direction



4. Illustration of Transformations on a plane

C is a reflection of A in the line x = 0 (the y-axis)
D is a rotation of A (0,0), 90 degrees clockwise
B is the translation of A by vector $\binom{6}{4}$



[The orange lines are just to help you see where the figures are on the 'graph paper']

Unit: Geometry	Topic: Circles
Benchmark	**9.2.2.12** Investigate angles formed by chords and tangent of a circle.

Learning Objectives: By the end of the topic, students will be able to;

- investigate angles formed by chords and tangent of a circle.

Essential questions:

- What is needed to understand angles formed in a circle by chords and tangents?

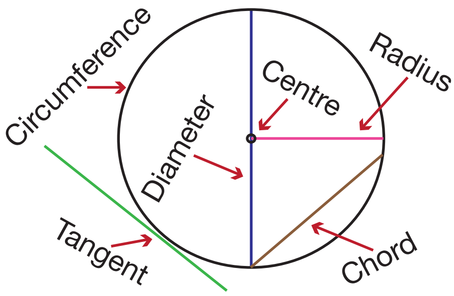
Key Concepts (ASK-MT)	
Attitudes/Values	• Confidence, Rationality, Participatory in finding angles formed by chords and tangents.
Skills	• Investigate and evaluate angles formed by chords and tangents.
Knowledge	• Circle, chords, Tangents, Angles formed.
Mathematical Thinking	• Think about and reason how angles are formed by chords and tangents of a circle.

Content Background

Angles formed by chords and tangents

Parts of a Circle

A circle is a special figure which has parts that have special names which have special angles lines and line segments that are exclusive to circles.
It is a plane figure with a center point with all lengths from the center point is all equal.
Centre: fixed one point called the center of the circle
Radius: Any segment with one end point at the center of the circle and the other end point on the circle is called a radius (plural is radii)
Chord: Any segment with its endpoints lie on the circle is a chord
Diameter: Any chord that passes through the center of the circle
Tangent: A straight line or a plane that touches a curve or curved figure at a point, but if extended does not cross it at that point
Circumference: the enclosing boundary of a curved geometric figure especially the circle.



1. Angles formed by Chords

How to find the measure of an angle formed by the two chords

Step 1: find the value of the two given arcs
Step 2: Find the measure of the supplemental angle
Step 3: subtract the supplemental arc from 180°

Supplemental angle = $\frac{\text{arc AB+arc CD}}{2}$

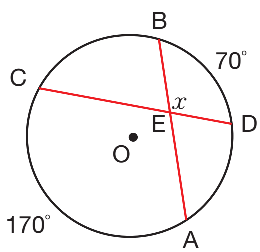
Supplemental angle = $\frac{60 + 33}{2}$

Supplemental angle = $\frac{93}{2}$

Supplemental angle = 46.5°

$\angle$AEC = 180 – 46.5

$\angle$AEC = 133.5°



$\angle BED$ is formed by two intersecting chords.
Its *intercepted arcs* are $\overset{\frown}{BD}$ and $\overset{\frown}{CA}$.
[Note: the intercepted arcs belong to the set of vertical angles.]

$m\angle BED = \frac{1}{2}(70 + 170) = \frac{1}{2}(240) = 120^\circ$

also, $m\angle CEA = 120^\circ\,(vertical\ angle)$

$m\angle BEC$ and $m\angle DEA = 60^\circ$ by a straight line

2. Angle formed inside of a circle by two intersecting Chords

When two chords intersect 'inside ' a circle, four angles are formed. At the point of intersection, two sets of vertical angles can be seen in the corners of x that are formed in the picture.

Remember: vertical angles are equal.

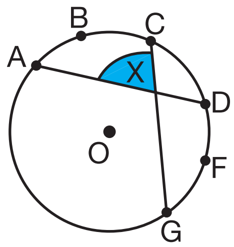
Angle formed inside of a circle = $\frac{1}{2}$ Sum of intercepted Arcs

Once you have found one of these angles, you automatically know the sizes of the other three angles by using your knowledge of vertical angles (being equal) and adjacent angles forming a straight line (adding to 180º)

<BED is formed by two intersecting chords

Therefore the formula for angles of two intersecting angles is;

$$m\angle x = \frac{1}{2}(\widehat{ABC} + \widehat{DFG})$$



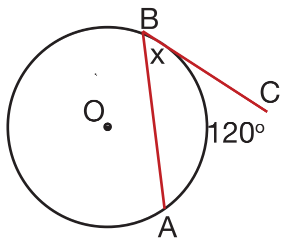
3. Angles formed by Tangents

Tangent Chord Angle

An angle formed by an intersecting tangent and a chord has its own vertex 'on 'the circle

Tangent Chord Angle = $\frac{1}{2}$ intercepted Arc

m$\angle$ABC= $\frac{1}{2}$ mAB



$\angle$ABC Is an angle formed by a tangent and chord.
Its intercepted arc is the mirror arc from A to B.

m$\angle$ABC = 60 °

Strand 3: Patterns and Algebra

Content Standard: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyze mathematical situations and collect, organize, and represent data to answer questions.

Units	Benchmark	Topics	Lesson Titles
Linear Functions	**9.3.3.1** Determine the slope and equation of a line when given the graph of a line, two points on the line, or the equation of the line.	Equation of a straight line	• Gradient of a straight line (y = mx+c) • Equation given the gradient and y-intercept • Equation given two points • Equation given one point and the gradient • Equation given the graph
	9.3.3.2 Solve and sketch linear equations.	Graphs of Linear equation	• Solve Linear equations • Plotting and sketching linear equations
	9.3.3.3 Apply and interpret linear relation modeling practical situations.	Linear Relations	• Interpret linear relationship between two quantities • Graph the linear relationship between two quantities
Equations and Inequality	**9.3.3.4** Solve linear simultaneous equations, using algebraic and graphical techniques including usage of digital technology.	Linear simultaneous equations	• Linear equations • Solving simultaneous equations using substitution method • Solving simultaneous equations using elimination method • Solving simultaneous equations using graphs and digital technology • Solve word problems involving simultaneous equations
	9.3.3.5 Investigate and solve single variable equations and inequalities using rational numbers and use number line to graph the solutions.	Linear equations and inequalities	• Basic rules for solving inequalities • Solving Inequalities with single variables • Representing Inequalities on a Number Line • Solve Linear Inequalities and represent the solutions set on a number line
Number Patterns	**9.3.3.6** Represent a variety of patterns, including recursive patterns, with tables, graphs, words and symbolic rules.	Patterns, sequences and formulae	• Completing and describing number patterns • Finding terms and writing rules for sequences • Using the Tn formula
	9.3.3.7 Identify and Apply appropriate mathematical formulae to find values of various patterns.	Formulae and Patterns	• Identify formulae for various patterns • Apply formulae to find values of various patterns • Number pattern problems • Solve problems of various patterns
	9.3.3.8 Solve various problems on number patterns.	Number pattern problems	• Problems of number patterns

Algebra	**9.3.3.9** Represent mathematical situations as algebraic expressions and equations and factorize by common factor.	Algebraic equations and factorization	• Formulate Algebraic expressions • Formulate Algebraic equations • Factorise algebraic expressions
	9.3.3.10 Factorize monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts.	Factorisation of Monic and non-monic quadratic expressions	• Factorise Monic quadratic expressions • Factorise non-monic quadratic expressions • Solve quadratic equations
	9.3.3.11 Apply the distributive law to the expansion of algebraic expression including binomially and collect like terms where appropriate.	Expansion of Binomial products	• Simplify algebraic expressions • Algebraic products • Algebraic quotients
	9.3.3.12 Simplify algebraic products and quotients using index laws.	Algebraic fractions	• Addition and subtraction of algebraic fractions • Multiplication and division of algebraic fractions • Word problems on algebraic fractions
	9.3.3.13 Apply the four operations to simple algebraic fractions with numerical denominations.	Simplify using Index laws	• Algebraic products using index laws • Algebraic quotients using index laws
	9.3.3.14 Substitute value into formulas to determine an unknown.	Substitution	• Substitute into formulae and determine the unknown • Substitute into the subject of the formulae to determine the solutions

Unit: Linear Functions **Topic: Equation of a Straight Line**

Benchmark	**9.3.3.1** Determine the slope and equation of a line when given the graph of a line, two points on the line, or the equation of the line.

Learning Objectives: By the end of the topic, students will be able to;

- identify and sketch a straight line with the corresponding pairs of (x,y) values and define graph of linear function (y=mx +c),
- define and conclude the equation of a straight line with y-intercept and gradient, and
- define and conclude the equation given two points, one point and the gradient and given the graph.

Essential questions:

- What information is required to determine the slope and equation of a straight line on a graph?
- What tools are used to construct a straight line graph

Key Concepts (ASK-MT)

Attitudes/Values	• Appreciate the construction of straight line graphs and critically evaluate the equations of the straight line graphs.
Skills	• Construct straight line graphs and determine their equations line. • Evaluate and draw conclusions to the equation of a straight line.
Knowledge	• Gain understanding of corresponding x, y values, linear function y = mx + c, m≠0, gradient = rise over run (difference of the coordinates or points on the line), y intercept, straight line equation, y = ax + b. • Assimilate the equations of the straight line graphs as a linear function.
Mathematical Thinking	• Think about, reason and communicate the conclusions to the equations of the straight line using gradient, corresponding points and y intercept.

Content Background

1. Linear function

When y is a function of x and is expressed as x, and is expressed as a linear expression of x ; y = m x + c, where m ≠ 0, we call this a linear function of x; e.g. $y = \frac{1}{3}x + 1$, where $m = \frac{1}{3}$ and c = 1 as the y intercept, when x = 0.

That is no matter how much the value of x increases, the average rate of change of the value of y corresponding to the value of x is constant and is equal to m.

Examples:

When y = 4x,
Then on the table it will read;

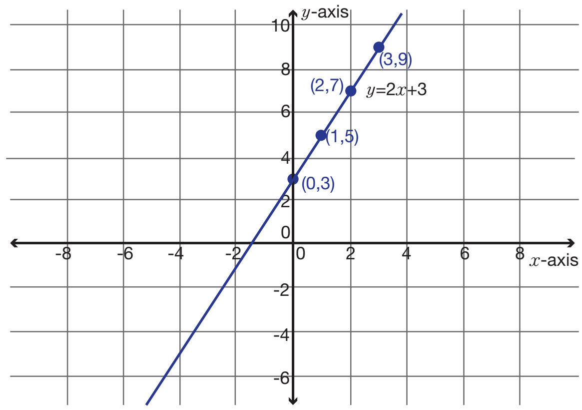
x	1 2 3 4...
y	4 8 12 16 ....

When y = 2x + 2
Then on the table it will read;

x	y
1	4
2	6
3	8
4	1
.	.
.	.

2. Equation of a straight line

The constant of the function and the y intercept determines the slope or gradient of the straight line and gives the equation of the line.



The gradient of the straight line is the difference of the two points (coordinates of x, y) on the line and that is calculated as rise over run which is $\frac{y - y1}{x - x1} = m$

An equation of a straight line is determined by the coordinating points of x, y values which are usually represented on table of values and a graph. They show also the relationship between the two variables by the coordinates that determine the straight line, the slope and y intercept.

What will be gradient of this line graph? gradient= $\frac{rise}{run} = \frac{4}{2} = 2$
What is the y-intercept in this line graph?
To find y-intercept, let x = 0
y = 2 x 0 + 3,
y = 3

Therefore y-intercept is 3 and 2 is the slope of the line.
The coordinates of the line graph satisfies the equation y= 2x + 3

Unit: Linear Functions | **Topic: Graphs of Linear Equation**

Benchmark	**9.3.3.2** Solve and sketch linear equations.

Learning Objectives: By the end of the topic, students will be able to;

- solve linear equations,
- sketch graphs of linear equations, and
- evaluate linear equations.

Essential questions:

- What information is used to solve linear equations?
- What content and skills are required to sketch graphs of leaner equations?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be creative and confident in solving and constructing graphs of linear equations critically and participatory with peers.
Skills	• Solve linear equations and sketch graphs of linear equations using the proper materials and tools.
Knowledge	• Gain understanding of Corresponding x, y values, gradient of the line and y intercept and use to solve and sketch graphs of linear equations.
Mathematical Thinking	• Think about, reason, speculate and communicate ways of solving and constructing linear equations.

Content Background

1. Solving Linear Equations

Linear equations are equations of the form (or can be simplified in the form) $ax+b=0$, where a and b are constants and x is the unknown or the variable

Example 1:

(a) $7x - 9 = -5$

$\therefore 7x - 9 + 9 = -5 + 9$

$7x = 4$

$7x \div 7 = 4 \div 7$

$\therefore x = \frac{4}{7}$

b) $17 = 8 - 4x$

$\therefore 17 - 8 = 8 - 8 - 4x$

$9 = -4x$

$9 \div -4 = -4x \div -4$

$-\frac{9}{4} = x$

$\therefore x = -\frac{9}{4}$

Example 2:

Solve

$\frac{m}{3} - 5 = -2$

$\therefore \frac{m}{3} - 5 + 5 = -2 + 5$

$\frac{m}{3} = 3$

$\frac{m}{3} \times 3 = 3 \times 3$

$\therefore m = 9$

Example 3:

If $y = 5x - 3$, find x when $y = -18$

$y = 5x - 3$

$\therefore -18 = 5x - 3$ (substitute the value for y)

$-18 + 3 = 5x - 3 + 3$ (add 3 to both sides)

$-15 = 5x$ (divide by 5)

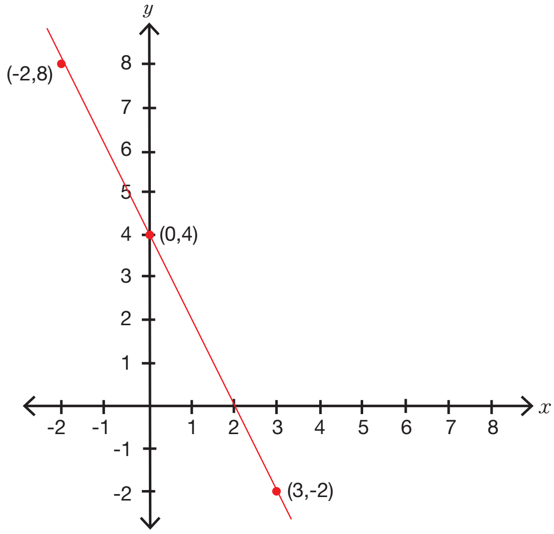
$-3 = x$

$\therefore x = -3$

2. Graphs of Linear equation

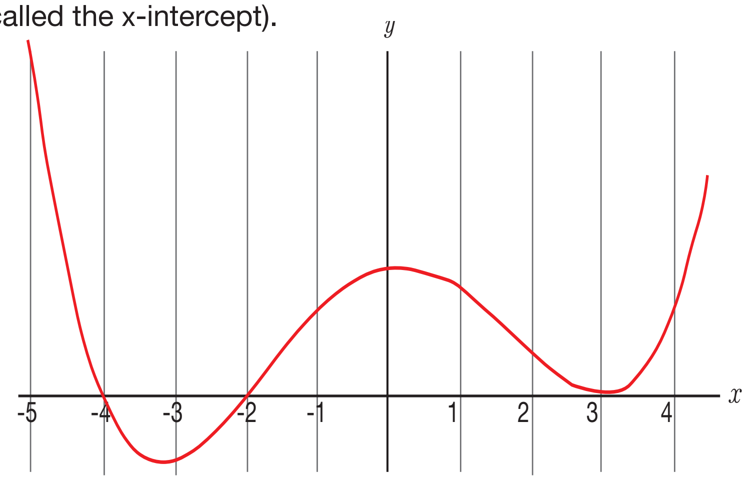
Sketching of Linear equations

All it takes to draw a straight line is to know any two points on the line. E.g. if (3,-2) and (-2, 8) are the coordinates of two points on the straight line and can be drawn.



A sketch graph is use to show the shape of a graph and its important points. It is not drawn to scale but should be drawn neatly.

The most important points for any graph are the points where the graph cuts the vertical axis (called y-intercept) and the horizontal axis (called the x-intercept).

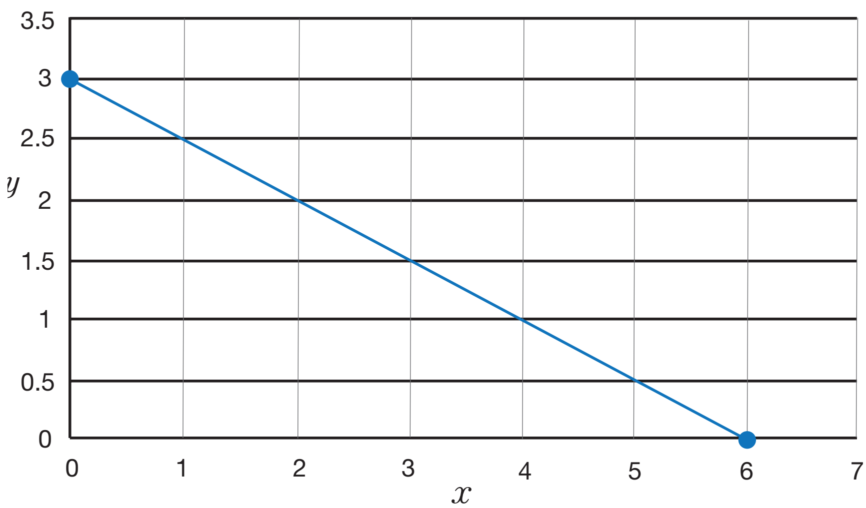


Examples:

Sketch each of the straight lines by finding the y intercept and the x- intercept.

x + 2y = 6 (To find the x -intercept, let y =0)
x + 2 x 0 = 6
x = 6, so x -intercept is (6,0)
x + 2y = 6 (to find the y-intercept let x =0)
0 + 2y = 6
2y = 6 , y = 3, so y intercept is (0,3)

Graph for x + 2y = 6

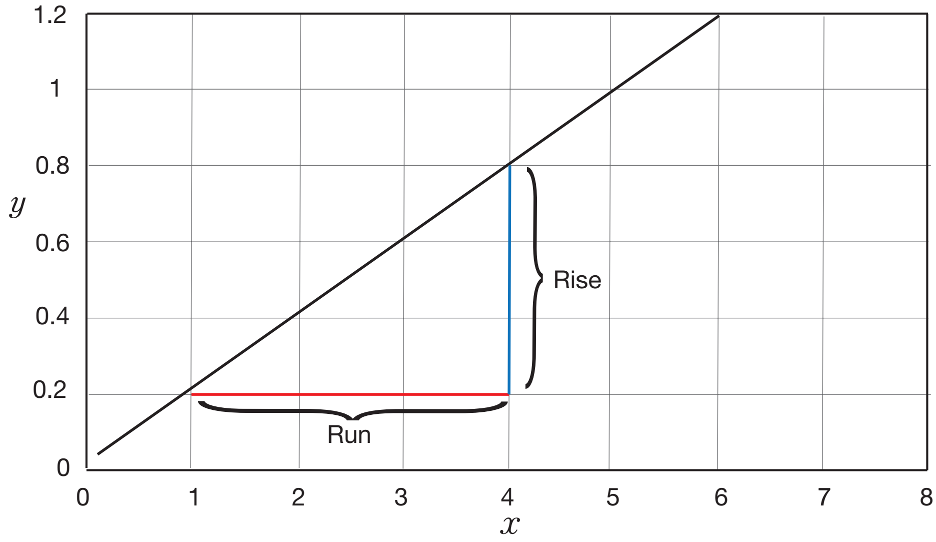


3. Evaluation of linear equation

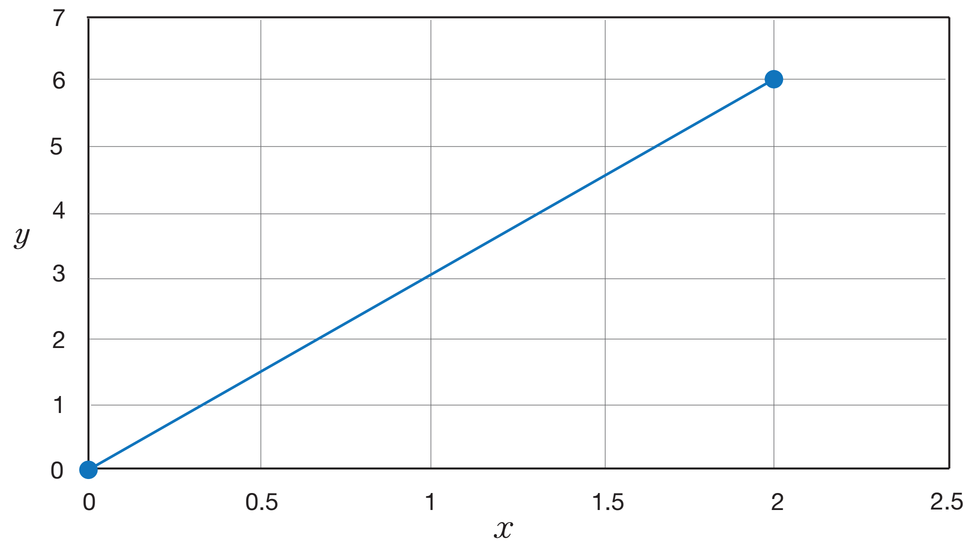
Linear equation is evaluated by a gradient of a straight line which is a measure of how steep the line is.

Bike riders feel the steepness of a hill as they find it hard to push the peddles. On the flat land we say that the gradient is zero. The gradient increases as the steepness of the road increases.

Therefore $Gradient = \frac{Vertical\ distance}{Horizontal\ distance}$ or $\frac{rise}{run}$



The gradient of a straight line is calculated by rise divide by run ($\frac{rise}{run}$)



Example 1:

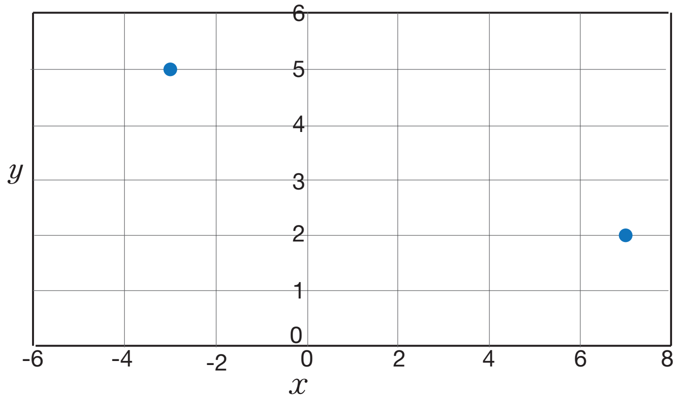
Find the gradients of the straight lines made by joining each of the following pairs of coordinates;

A (0,0) B (2,6) Gradient = $\frac{6}{2} = 3$

Example 2:

Plot the points A (-3,5) and B (7,2)

a) Find the gradient of the lines through A and B



b) Find the lengths of the triangle. The slope is negative as it is downhill;

Gradient = $\frac{3}{10}$

Summary:

To find the x intercept let y = 0

To find the y intercept, let x = 0

Gradient = $(\frac{rise}{run})$

The gradient is positive, if the line slopes upwards from left to right.

The gradient is negative, if the line slopes upwards from right to left.

Unit: Linear Functions | **Topic: Linear Relations**

Benchmark	**9.3.3.3** Apply and interpret linear relation modeling practical situations.

Learning Objectives: By the end of the topic, students will be able to;

- interpret linear relationship between two quantities in practical situations, and
- graph the linear relationship between two quantities.

Essential questions:

- What skill and process is required to apply and interpret the linear relation in practical situations?
- How are linear relations important in practical situations?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate linear relations in everyday situations and consider the usefulness of linear relations of various quantities in practice.
Skills	• Interpret linear relationships between two quantities • Create linear relations between two quantities and graph on table of values and the line graph.
Knowledge	• Evaluate and understand linear relationship between two quantities as represented *e.g.* $y = 2x + 3$ and $y = x + 5$ and how important they are in practice.
Mathematical Thinking	• Reason, apply and communicate ways and ideas of how to represent 2 or more quantities on linear graphs and tables.

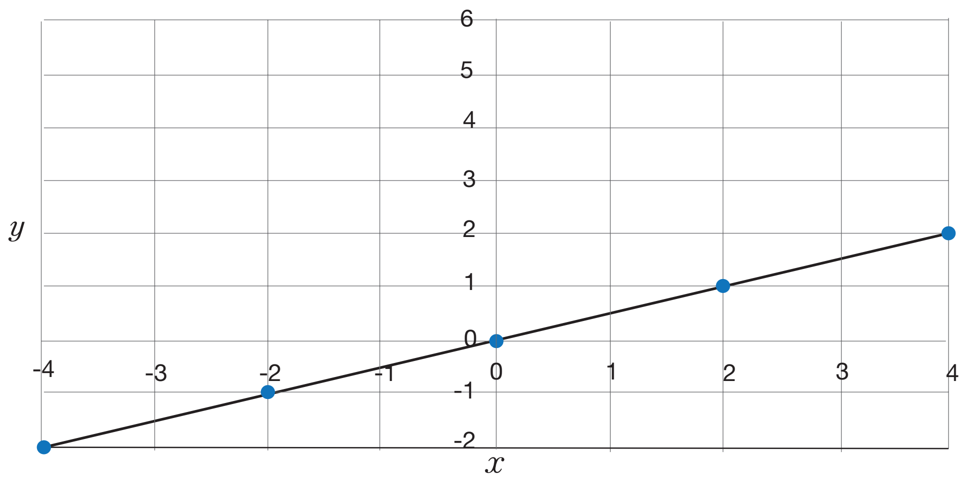
Content Background

Graphing Linear Relations
Consider all points (x,y) in which y = 2x. We can write down any ordered pairs that satisfy this rule. For example (1, 2), (2, 4), (3, 6), (-2,-4), (0, 7), (1, 4)

To visualize all the points that satisfy the equation y = 2x , we draw a graph of all points that near the origin, 0. Often we find that a table of values is useful.

For y = 2x the table of values is;

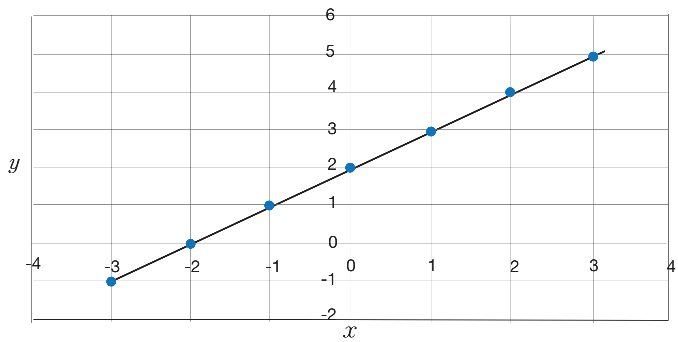
x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6



Draw the lines of graphs with these equations

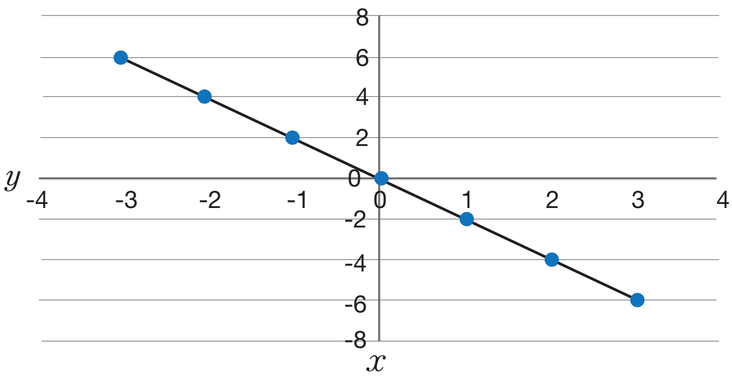
a) y = x + 2

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5



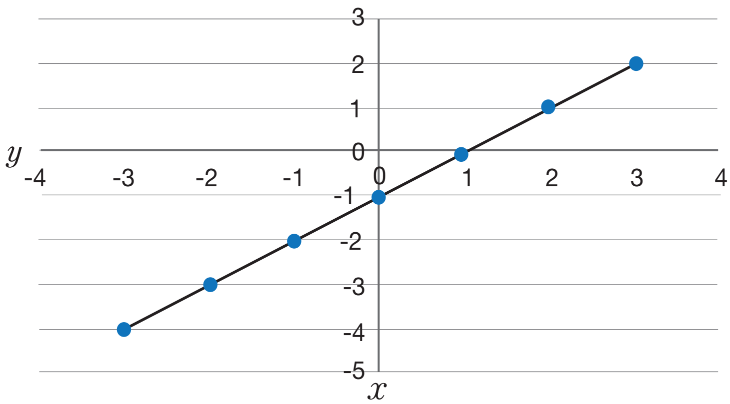
b) y = -2x

x	-3	-2	-1	0	1	2	3
y	+6	+4	+2	0	-2	-4	-6



c) y = x - 1

x	-3	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1	2



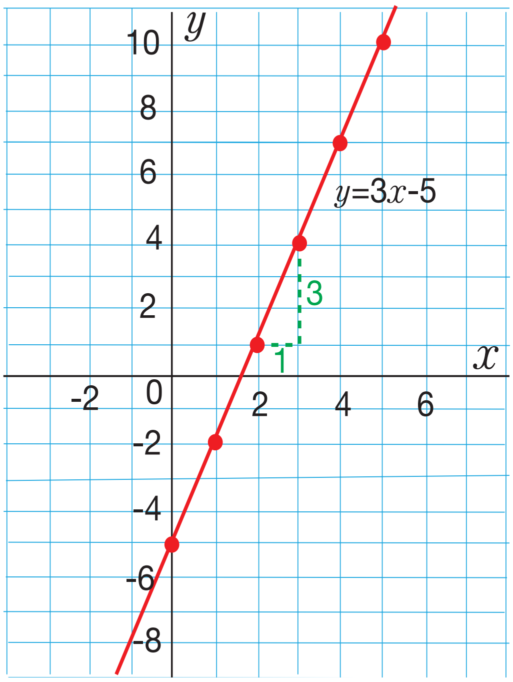
Linear relations are patterns of quantities that show the relationship between two or more quantities. These can be shown on the tables of values of the comparison of two values where x value changes, the y value also changes thus patterns of relationships are formed to help with daily life decisions.

For example, shown the table of values of sending text messages of 20t per message from the Flex card that you buy daily. What pattern can you see on the table and if shown on the graph. How will this pattern affect your decision on flex card top ups?
Represent on the table of values and the graph showing the relationship between two quantities, when x is the number of messages and y is the cost per message.

Example:

Show the table of values on the line graph

x	0	1	2	3	4	5
y	-5	-2	1	4	7	10



Unit: Equations and Inequality	Topic: Linear Simultaneous Equations
Benchmark	**9.3.3.4** Solve linear simultaneous equations, using algebraic and graphical techniques including usage of digital technology.

Learning Objectives: By the end of the topic, students will be able to;

- calculate and solve Linear equations,
- solve simultaneous equations using substitution, elimination, graphical and digital technology, and
- solve word problems involving simultaneous equations.

Essential questions:

- What is needed to solve linear simultaneous equation?
- What methods will be used and how will each method be used?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be appreciative and confident with use of substitution and elimination methods to solve linear equations. Participate critically with peers.
Skills	• Compare and apply calculation of linear equations using substitution and elimination methods, also graphs and digital technology.
Knowledge	• Use previous knowledge to understand linear simultaneous equation, substitution method, elimination method, and use graphical techniques of solving, use of digital technology.
Mathematical Thinking	• Use previous knowledge to reason and explain substitution and elimination methods calculation processes.

Content Background

Simultaneous linear equation

To solve a pair of Simultaneous linear equation in x and y, we need to find a pair of values of x and y that satisfy both the given equations at the same time. There are two algebraic methods of solving the Simultaneous linear equation;

- substitution method
- elimination method

Solution of simultaneous linear equation by *substitution method*;

- Make x or y the subject of one equation and substitute into the other equation
- Solve the resulting equation
- Substitute the value back into one of the original equations to find the value of the other pronominal
- Check that your answers satisfy the original equations

Example;

$3x + 4y = 11$

$y = 9 - 2x$

Solution

$3x + 4y = 11$.............................(1)

$y = 9 - 2x$ ....(2)

Substitute (2) into (1)

$3x + 4(9 - 2x) = 11$

$3x + 36 - 8x = 11$

$-5x = 11 - 36$

$-5x = -25$

$x = 5$

Then substitute $x = 5$ into (2), $y = 9 - 2(5)$

$y = -1$

Therefore the solution is $x = 5$ and $y = -1$

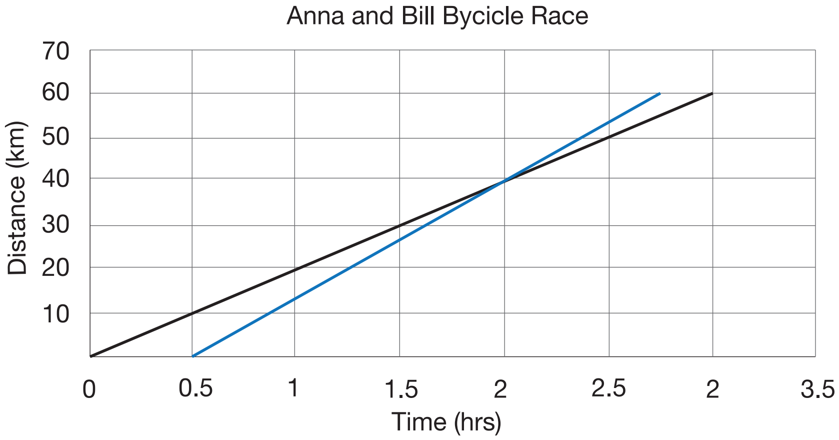
The *elimination* method is used to create an equation involving just one pronominal, which can then be solved. Substitution gives the value of the other pronominal.

Example:
Solve for x and y

$2x + y = 8$ (1)
$x - y = 1$ (2)

Solution:
We can eliminate the pronominal by adding the left hand sides and the right hand sides of the two equations
+ (2) $(2x + y) + (x - y) = 8 + 1$
$2x + y + x - y = 8 + 1$ (collecting like terms)
$3x = 9$
$x = 3$
Substitute $x = 3$ into (1) to find the y value
$2 \times 3 + y = 8$
$6 + y = 8$
$y = 2$
Check mentally that solution (3, 2) satisfies the equation (1) and (2)

Solving simultaneous equations graphically
Example 1:
Anna and Bill had a bicycle race. Anna left before Bill, who caught up after 2 hours and went on to win the race.
Here is a graph showing the distance (measured in kilometres) each travelled over time (measured in hours)



The point where the lines cross or intersect represents the time and the distance where Bill catches up to Anna. The coordinates of this point of intersection are (2hrs, 40km)
Example 2: Problem solving

1. I am thinking of 2 whole numbers. Their sum is 20 and their difference is 6.
   What are the numbers?
   1. $x + y = 20$
   2. $x - y = 6$ (Add the equations to find the values of 2 numbers)
   $2x = 26$
   $x = 13$ (Substitute into 1 to find value of y)
   $13 + y = 20$
   $y = 7$

   The two numbers are 13 and 7 (point of intersection)

2. I am thinking of 2 whole numbers. Their sum is 10, and if you add the 1st number to double the 2nd number, the answer is 8. What are the numbers?

1. $x + y = 10$
2. $2x + y = 8$ (subtract the equations to find the values of x and y)

$-x = 2$

$x = -2$ (Substitute into 1 to find value of y)

$y - 2 + y = 10$

$y = 12$

The two numbers are -2 and 12 (point of intersection)

3. I am thinking of 2 whole numbers. Their sum is 15, and the 1st number is double the 2nd number. What are the numbers?

$x + \frac{1}{2}y = 15$

To find y , let x = 0

$\frac{1}{2}y = 15$, then $y = 30$

$x + \frac{1}{2} \times 30 = 15$ (Substitute to find value of x)

$x = 0$

The two numbers are 0 and 15 (point of intersection)

Unit: Equations and Inequality	Topic: Linear Equations and Inequalities
Benchmark	**9.3.3.5** Investigate and solve single variable equations and inequalities using rational numbers and use number line to graph the solutions.

Learning Objectives: By the end of the topic, students will be able to;

- apply the basic rules for solving inequalities, and
- solve inequalities with single variables.

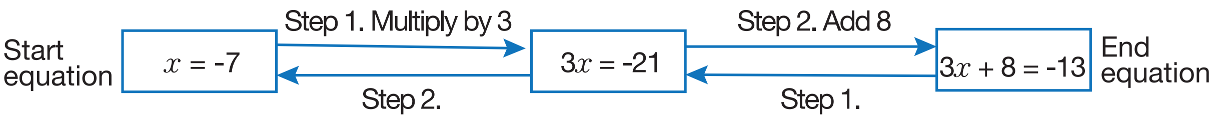
Essential questions:

- What is required to solve and graph single variable equations and inequalities?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be diligent and confident in applying rules to solve inequalities working cooperatively with peers.
Skills	• Investigate and solve inequalities of one variable using the basic rules.
Knowledge	• Gain understanding of single variable equations ($ax + b = 0$), a and b are constants, x is the unknown variable, symbols and meaning of inequalities >, <, ≤, ≥, rules for inequalities using the 4 operations, positive and negative numbers, graphing on number line.
Mathematical Thinking	• Reason and apply rules following rules and communicating the ideas.

Content Background

Equation and inequality



Example:

Build, $x \longrightarrow 3x \longrightarrow 3x + 2 \longrightarrow 3x + 2 \geq -7$,

then solve for x :

$3x + 2 \geq -7 \longrightarrow 3x + 2 - 2 \geq -7 - 2 \longrightarrow 3x \geq -9 \longrightarrow 3x \div 3 \geq -9 \div 3 \longrightarrow x \geq -3$

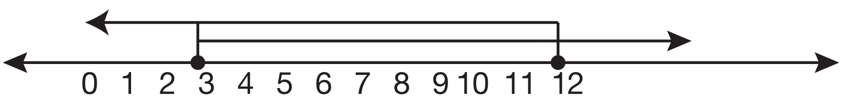
(Maintain the inequality sign and use inverse operation to find the value of the unknown x)

Simultaneous equation is about solving two linear equations or inequalities simultaneously to find the value of the unknown (x) to satisfy either equations or inequalities at the same time.
For example; consider the following linear inequalities

$x + 7 \leq 19$

$x - 1 \geq 2$

When we solve them, $x \leq 12$ from (1) and $x \geq 3$ from (2)
When represented on the number line the common solution between the solutions shown is;



The overlapping part on the line graph shows that the common solution of these two inequalities is $3 \leq x \geq 12$

Inequalities
We use an inequality to model a situation that can be described by a range of numbers instead of a single number. When one equality is less than or equal to another quantity, we use the symbol ;≤. When one equality is greater than equal to another quantity, we use the symbol; ≥

Examples of inequality statements;
a) a less than 3 ; $a < 3$
a) b greater than – 4 ; $b > -4$
c) c is less than and equal to $\frac{3}{4}$: $c \leq \frac{3}{4}$

Solving linear inequalities
Example 1:
Write an inequality to describe a situation. Define a variable and write an inequality to describe each situation. Determine the solution to find the unknown variable that satisfies the situation.
A contest entrant must be at least 18 years old
let a be the age of the contest entrant. So a can be 18 years or greater than 18 years old.
The inequality is $a \geq 18$

Example 2:
Determine whether a number is a solution of an inequality when given, you have to justify the answers. E.g. $b > -4$

Method 1:
Use a number line; show all the numbers on a line. The b ≥-4 is all the numbers greater than or equal to – 4. For the number to be greater than -4, it must lie to the right of -4.

Method 2:
Use substitution. Substitute each number for b in the inequality b ≥-4
Determine whether the resulting inequality is true or false.
e.g. 1. Since -8 ≥ -4 is false, - 8 is not a solution
e.g. 2 Since -3.5 ≥ -4 is true, - 3.5 is a solution

Example 3:
Graph Inequalities on a Number Line
a) Graph the inequality on the number line and
b) Describe the number line
c) Find possible solutions of the variable on the number line

E.g. $x > 3$ inequality

Graph 

Possible solutions 3, 3.5, 4.05, 8

Unit: Number Patterns	Topic: Patterns, Sequences and Formulae
Benchmark	**9.3.3.6** Represent a variety of patterns, including recursive patterns, with tables, graphs, words and symbolic rules.

Learning Objectives: By the end of the topic, students will be able to;

- completing and describing number patterns,
- finding terms and writing rules for sequences, and
- using the T_n formula.

Essential questions:

- How would number patterns be represented as recursive, on tables, graphs, words and as symbolic rules?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate critically the number patterns and participate responsibly in completing and describing number patterns.
Skills	• Examine patterns, sequence, recursive patterns and represent on graphs, tables, words. • Reason and find their symbolic rules and formulae.
Knowledge	• Understand patterns, sequence, recursive patterns, and how these can be represented on graphs, tables, words using symbolic rules and formulae.
Mathematical Thinking	• Reason and explain number patterns, sequence and recursive patterns and present meaningfully.

Content Background

Number Patterns
A list of numbers that follow a certain sequence
Example
1: 1, 4, 7, 10, 13, 16… starts at 1 and jumps every 3 times
2: 2, 4, 8,16, 32 … double each time

Patterns, sequences and formulae
Patterns are seen in forms of designs, decorations, shapes, forms, arrangements and configurations. Patterns are repetitive in forms or design and can be continuous or fixed. Sequences are orders, arrangements, classifications, categorizations, systems, structures, disarrays and series. Sequence is order in which patterns, forms, designs etc. are organized.
Formulae are formulations, formularies, methods, plans, recipes, prescriptions, procedures and principles. Formulae are ways or methods of which patterns, designs etc. are formed or arranged.

Completing and describing number patterns Investigations:

Number shapes
Consider the patterns made by dot shapes
1, 1 + 2 = 3, 2 + 3 = 5, 3 + 4 = 7, 4 + 5 = 9, - odd shapes
1, 4, 9 – square shaped numbers
1, 6,15 – hexagon shaped numbers

Perfect Numbers

A factor of a number is one that divides into the number exactly. The Greeks thought that numbers whose factors added to the numbers themselves were perfect numbers. For example those factors of 6 are 1, 2, 3 and 6=1+2+3

If the factors sum to a total smaller than the number, it is called deficient. For example, the factors of 8 are 1, 2 and 4 but 1 +2 + 4= 7

If the factors, add to more than the number itself it is called abundant. The factors of 12 are 1,2,3,4, and 6, but 1+2+3+4+6 = 16

Exercise:

a) Find the perfect, deficient and abundant numbers between 1 and 50
b) The two numbers 220 and 284 have special properties. Find the sum of the factors for each number and discover what it is.

Puzzling Numbers

Numbers that do not have factors other than one or themselves are called prime numbers.

Finding rules and writing rules for sequences

All numbers can be written as the product of prime factors with every number having a different set of prime factors

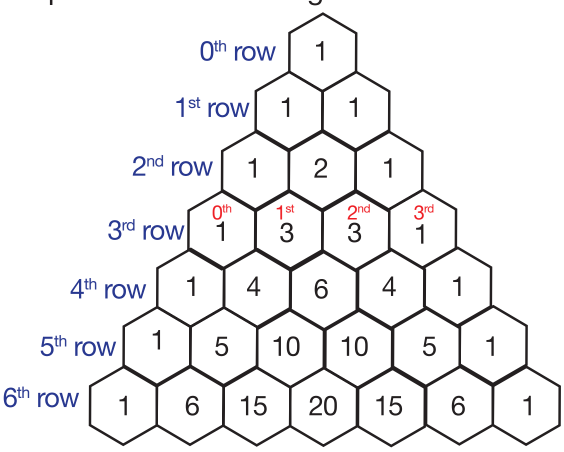
Example 1:

24 (2 x 12) – 12 (2 x 6) - 6 (2x3)- 3 (1x3), that is 24 = 2x2x2x3= 23 x 3

Example 2:

48150 (2 x 24255), 24255 (3 x 8085), 80855(3 x 2695), 2695 (5 x 539), 539 (7 x 77), 77 (7 x 11), 11 (1x 11) that is 48150 = 2 x 32 x 5 x 72 x 11

Example 3: Pascal's Triangle and its formulae



$$\binom{n}{1}=\binom{n}{0}\cdot\frac{(n-1+1)}{1}=\frac{n}{1}$$

$$\binom{n}{2}=\binom{n}{1}\cdot\frac{(n-2+1)}{2}=\frac{n}{1}\cdot\frac{n-1}{2}$$

$$\binom{n}{3}=\binom{n}{2}\cdot\frac{(n-3+1)}{3}=\frac{n}{1}\cdot\frac{(n-1)}{2}\cdot\frac{(n-2)}{3}$$

$$\binom{n}{4}=\binom{n}{3}\cdot\frac{(n-4+1)}{4}=\frac{n}{1}\cdot\frac{(n-1)}{2}\cdot\frac{(n-2)}{3}\cdot\frac{(n-3)}{4}$$

Using the T_n Formula

Examples

T_n for number sequence 3, 5, 7, 9, 11… n

$T_n = a + (n-1)d$

$T_n = 3 + (n-1)2$

$T_n = 3 + 2n-2$

$T_n = 2n -1$

$T_1 = 2 \times 1 – 1 = 1$

$T_2 = 2 \times 2 – 1 = 3$

$T_3 = 2 \times 3 -1= 5$

Therefore the formula for the number sequence is $T_n = 2n -1$

T_n for number sequence 1, 6, 11, 16, 21…n

T_n for number sequence 1, 6, 11, 16, 21…n

$T_n = 1 + (n-1)5$

$T_n = 1 + 5n-5$

$T_n = - 4 + 5n$

$T_n = 5n -4$

$T_1 = 5 \times 1 - 4$

$T_1 = 1$

$T_2 = 5 \times 2 - 4$

$T_2 = 6$

$T_3 = 5 \times 3 -4$

$T_3 = 15-4$

$T_3 = 11$

The formula for the number sequence above is $T_n = 5n -4$

Unit: Number Patterns	Topic: Formulae and Patterns
Benchmark	**9.3.3.7** Identify and Apply appropriate mathematical formulae to find values of various patterns.

Learning Objectives: By the end of the topic, students will be able to;

- identify formulae for various patterns, and
- apply formulae to find values of various patterns.

Essential questions:

- How would you formulate mathematical formulae to find values of various patterns?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be motivated and interested in finding formulae and values of number patterns and critically participate in sharing ideas with others.
Skills	• Reflect from previous ideas, identify formulae for various patterns and find their values.
Knowledge	• Gain understanding about mathematical formulae of various patterns and how to identify patterns and formulate the formulae.
Mathematical Thinking	• Explain ways of identifying various patterns and thinking about formulating the formulae to find their values.

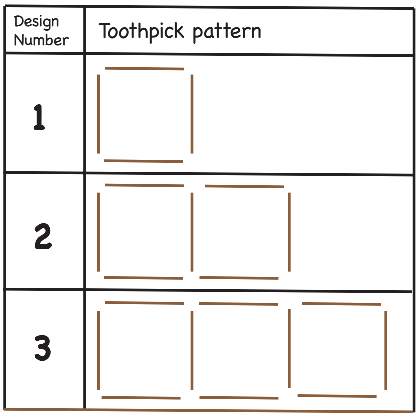
Content Background

Mathematical formulae for number patterns

Identifying Formula:

How do we generate equations or formulae from patterns?

Let's look at the Toothpick pattern



The number sequence looks like this (4, 7, 10, 13 …)

1. Students may have to work out the difference between two adjacent numbers and may say the difference is 3 and may guess that the pattern is a number multiply by 3; however a number is needed to get to the total number of toothpicks for each pattern. Let them find out.

2. Let students to make a table of the difference

Pattern (n)	Toothpicks (T)	3 x n
1	4	3
2	7	6
3	10	9
4	13	12
...	...	...
...	...	...
...	...	...

3. Then to form the toothpick pattern for the above square pattern, the students can easily work out the formula to be; T = 3n + 1

There can be other strategies that we can also use to find another formula that can arrive at the same number of toothpicks to make the pattern.

For example, if we make separate patterns we will need 4 toothpicks to make each pattern (n). The formulae will be T= 4n, however if we need to still keep the same pattern then we would use this formulae to create the pattern.
If T = 4n, then we need to subtract 1 to start the pattern of 4 tooth picks for the 1^{st} pattern (n-1)

That is;
T = 4n – (n-1),
Then for T_1 = 4 x 1 – (1-1)
= 4
For T_2 = 4 x 2 – (2-1)
= 8 – 1
= 7

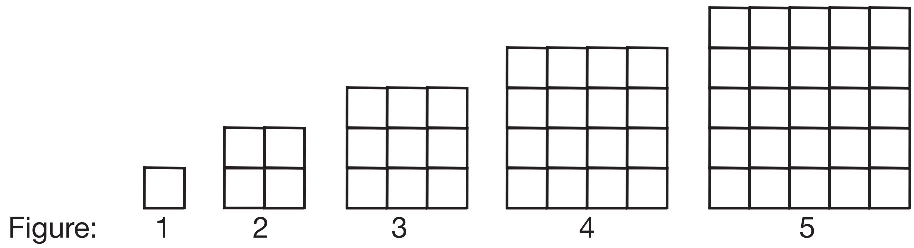
Then the formula can look like this too, T= 4n – n + 1, 4 x 1 – 1 + 1 = 4, 4 x 2 – 2 + 1 = 7 and so on.

Students can use this same idea to generate the formula for the same pattern or other patterns

Investigate and generate formulae for the number patterns below:

Start with counting numbers: 1,2,3,4 … (n + 1)
Odd numbers: 1, 3, 5,7, 9, 11…(n + 2)
Even numbers: 2, 4,6,8,10,12 … (n x 2)
Prime numbers: 1, 3, 7, 11, 13, 19, 21…n?

Other numbers patterns and formulate their formulae, then find the values



Figures	1	2	3	4	5	n
Tiles	1	4	9	16	25	n^2

Unit: Number Patterns	Topic: Number Pattern Problems
Benchmark	**9.3.3.8** Solve various problems on number patterns.

Learning Objective: By the end of the topic, students will be able to;

- solve problems of various number patterns.

Essential questions:

- What skill and process is required to solve number patterns?

Key Concepts (ASK-MT)	
Attitudes/Values	• Build confidence and be appreciative about solving number pattern problems critically and participate well with others.
Skills	• Investigate and solve various problems involving number patterns.
Knowledge	• Demonstrate understanding of the process of solving number patterns *e.g* patterns of addition, subtraction or multiplication.
Mathematical Thinking	• Reason critically about solving various problems of number patterns and communicate ideas with others.

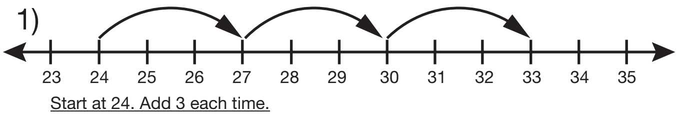
Content Background

Problems about Number Patterns

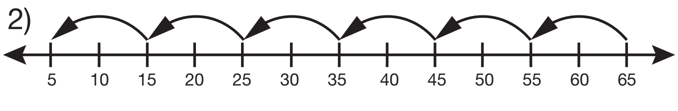
1. Making Number Patterns
Make number patterns for each of the rules
Start at 63 and subtract 4 each time ---- ----- ----- ----
Start at 1 and add 7 each time, ---- ----- ----- -----
Start at 17 and add 8 each time, ----- ----- ----- -----
Start at 68 and subtract 6 each time, ----- ----- ----- -----
And so on…

2. Recognizing Patterns
Describe the patterns for the hops on the number line

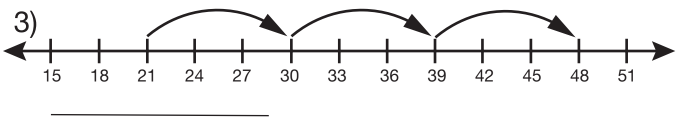
1) 24, 27, 30, 33,___, ___,___
(start at 24 and add 3 each time to hop to the next number)



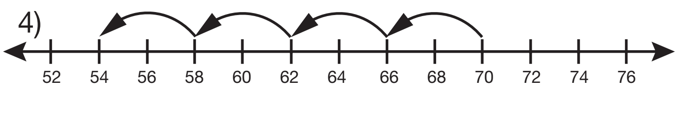
2) 65, 55, 45, 35, 25, 15, 5,___,___,___
(start at 65 and subtract 10 each time to hop to the next number)



3) 21, 30, 39, 45, ___,___,___,
(start at 21 and add 9 each time to hop to the next number)



4) 70, 66, 62, 58, 54, ___,___,___,
(start at 70 and subtract 4 each time to hop to the next number)



3. Identifying Number Pattern Rules

Work out the rules for each pattern, they may increase (addition) or decrease (subtraction)

14, 18, 22, ____, _____, _____, Rule: ____________________________

28, 26, 24, _____, _____, _____ , Rule:__________________________

45, ____, 75, 80, _____, _____, Rule:____________________________

150, 145, _____, 135, _____, _____, Rule: _______________________

90, _____, 96, 99, ____, ____, Rule_________________________

Patterns of multiplication and addition (making doggy numbers)

1x 9 +2 = 11

12 x 9 + 3 = 111

123 x 9 +4 = 1111

1234 x 9 +5 = 11111

12345 x 9 + 6 = 111111

123456 x 9 + 7 = 1111111

1234567 x 9 + 8 = 11111111

12345678 x 9 + 9 = 111111111

123456789 x 9 + 10 = 1111111111

Unit: Algebra	Topic: Algebraic Equations and Factorisation
Benchmark	**9.3.3.9** Represent mathematical situations as algebraic expressions and equations and factorize by common factor.

Learning Objectives: By the end of the topic, students will be able to;

- formulate Algebraic expressions from mathematical situations,
- formulate Algebraic equations, and
- factorize algebraic expressions.

Essential questions:

- How are mathematical situations represented as algebraic expressions and equations?
- How is factorization method used?
- What is a common factor?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be diligent and confident about factorisation of algebraic expressions and collaborate with others in formulating expressions and finding their factors of the expressions.
Skills	• Formulate algebraic expressions from situations and factorise with common factors.
Knowledge	• Have an understanding of algebraic expressions, algebraic equations, factorisation, common factors and how to formulate the expressions.
Mathematical Thinking	• Apply understanding of algebraic expressions and communicate ideas on how to factorise the expressions.

Content Background

1. Algebraic expression

An algebraic expression is an expression built up from integer constants, variables and the operations (addition, subtraction, multiplication and division and the exponentiation, that is a rational number). For example $3x^2 - 2xy + c$ is an algebraic expression.

2. Factorization

The process of expressing a polynomial as two or more polynomials is called factorization.
The polynomial $x^2 + cx + d$, where $a + b = c$ and $ab = d$, can be factorized into $(x + a)(x + b)$

For example; when we factor $x^2 + 3x + 2$ into $(x + 1)(x + 2)$, v + 1 and $x + 2$ are factors of $x^2 + 3x + 2$
Consider the identity $(x – 3)(x – 4) = x^2 – 7x + 12$. Going from left to right is called expansion and going from right to left is called factorization. Once a quadratic expression is factorised, it is easy to write down the solutions to the corresponding equation.

Example:

Factorize the following;

a) $4c^2 + 6c$ b) $– 4x + 8$ c) $x(x + 3y) – 2(x + 3)$

When you factorize the above 3 examples of the expressions, you will find the common factors of each expression for each term. The common factors for $4c^2 + 6c$ are 2 and c because 2 can divide into 4 and 6 and c into c^2 and c. therefore factors of $4c^2 + 6c$ are $2c$ and $2c + 3$, thus the factor of the expression is. $2c\ (2c + 3)$

Factorize $x^2 + 6x + 8$ and find its common factors.

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

Unit: Algebra	Topic: Factorization of Monic and Non-monic Quadratic Expressions
Benchmark	**9.3.3.10** Factorize monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts.

Learning Objectives: By the end of the topic, students will be able to;

- factorize Monic quadratic expressions,
- factorize non-monic quadratic expressions, and
- solve quadratic equations.

Essential questions:

- How is factorization method used to factorize monic and non-monic quadratic expressions?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be passionate about monic and non-monic quadratic expressions and critically factorizing the expressions.
Skills	• Analyse and factorise monic and non-monic quadratic expressions.
Knowledge	• Know the process of factorisation and be able to understand how to factorise monic and non-monic quadratic expressions.
Mathematical Thinking	• Reason and apply accurately the method of factorisation process for monic and non-monic quadratic expressions.

Content Background

1. Polynomial expressions

A polynomial is an expression such as $x^5 - 5x^2 + 7x$, $3x^7 + 2$ and $\frac{1}{5}x^2 + 2x - 5$
A polynomial may have any number of terms (the word 'polynomial' means many terms), but each term must be multiple of a whole-number power of x.
The term of highest index amongst then non-zero terms is called the leading term. Its coefficient is called the leading coefficient, and its index is called the degree of the polynomial. Thus

- $x^5 – 5x^2 + 7x$ has the leading term x^5, and leading coefficient 1 and degree 5
- $3x^7 + 2$ has leading term $3x^7$, leading coefficient 3 and degree 7
- $15x^2 + 2x – 5$ has leading term $15x^2$. Leading coefficient 15 and degree 2.

A monic polynomial has leading coefficient one such as $x^5 – 5x^2 + 7x$.The other two examples are non-monic because neither leading coefficient is 1.

A polynomial of degree 2 with leading coefficient 1 such as $x^2 – 6x + 2$, are called quadratic (or monic quadratic)
A polynomial of degree 2 with leading coefficient other than 1 such as $ax^2 - bx + c$, are called quadratic trinomials (or non-monic quadratic), where $a \neq 1$

2. Factorization of monic and non-monic quadratic expressions

A monic quadratic expression is of the form $ax^2 + bx + c$, where a, b and c are constants and $a =1$.
We can factorize monic quadratics expressions.

Example: Factorize

a. $x^2 + 5x + 6$ b. $x^2 + 7x + 10$ c. $x^2 + 7x + 12$

Solutions

a. $x^2 + 5x + 6$ Two numbers that add to give 5 and whose product is 6 are 3 and 2
$\therefore x^2 + 5x + 6 = (x+3)(x+2)$

b. $x^2 + 7x + 10$ Two numbers that add to give 7 and whose product is 10 are 5 and 2.
$\therefore x^2 + 7x + 10 = (x+5)(x+2)$

c. $x^2 + 7x + 12$ Two numbers that add to give 7 and whose product is 12 are 4 and 3.
$\therefore x^2 + 7x + 12 = (x+4)(x+3)$

A non-monic quadratic equation is of the form $ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$. There are many ways to factorize non-monic quadratic expressions. One of those ways is shown below using the three examples.

Example: Factorize

a. $2x^2 + x - 3$ b. $3x^2 + 16x + 5$ c. $5x^2 + 13 - 6$

Solutions

a. $2x^2 + x - 3$ $2x - 3 = -6$ (2 is the coefficient of $2x^2$,the first term and -3 is the constant last term)

$= \frac{(2x+3)(2x-2)}{2}$ $\therefore$need two numbers with a product of -6 and a sum of +1. These are 3 and -2.

$= (2x + 3)(x - 1)$

b. $3x^2 + 16x + 5$ $3 \times 5 = 15$

$= \frac{(3x+15)(3x+1)}{3}$ $\therefore$ need two numbers with a product of 15 and a sum of 16. These are 15 and 1.

$= (2x+3)(x-1)$

c. $5x^2 + 13x - 6$ $5x - 6 = -30$

$= \frac{(5x+15)(5x-2)}{5}$ $\therefore$ need two numbers with a product of -30 and a sum of +13. These are 15 and -2.

$= (x+3)(5x-2)$

Note: Place the coefficient of x^2 together with x at the beginning of each bracket and divide the whole expression by this coefficient to maintain equality.

Unit: Algebra	Topic: Binomials
Benchmark	**9.3.3.11** Apply the distributive law to the expansion of algebraic expression including binomials and collect like terms where appropriate.

Learning Objectives: By the end of the topic, students will be able to;

- simplify algebraic expressions,
- define binomials and binomial products, and
- expand binomial products of expression.

Essential questions:

- How is distributive law used to expand the expressions?
- What are binomials?
- How are they expanded?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be appreciative and confident in using the distributive law to expand algebraic expressions and binomial product expressions critically.
Skills	• Expand algebraic expressions, including binomials and collect like terms of the expressions.
Knowledge	• Gain understanding of the distributive law of expansion, binomial expressions, binomial product and how to apply in the expansions of the algebraic and binomial product expressions.
Mathematical Thinking	• Think about and generalise how to apply the distributive law in the expansion of algebraic expressions with reasoning.

Content Background

1.Expansion and Simplifying algebraic expressions

Examples: Expand and simplify by collecting like terms

a. $(x+3)(x-5)$ b. $(x-2)(x-1)$ c. $(x-5)(x+7)$

Solutions

a. $(x+30)(x-5)$
$= x(x-5)+3(x-5)$
$= x^2 - 5x + 3x - 15$
$= x^2 - 2x - 15$

b. $(x-2)(x-1)$
$= x(x-1) - 2(x-1)$
$= x^2 - x + 2x + 2$
$= x^2 - 3x + 2$

c. $(x-5)(x+7)$
$= x(x+7) - 5(x+7)$
$= x^2 + 7x - 5x - 35$
$= x^2 + 2x - 35$

2. Binomial products

Expansion of binomial products of expressions with perfect squares

$(a+b)^2 = (a+b)(a+b)$
$= a(a+b) + b(a+b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$,

So, $(a+b)^2 = a^2 + 2ab + b^2$,

And similarly $(a-b)^2 = a^2 - 2ab + b^2$

Examples: Expand the following perfect squares

a) $(x+5)^2$ b) $(y-3)^2$ c) $(4x-5)^2$

Solutions

a) $(x+5)^2$
$= x^2 + 2 \times x \times 5 + 5^2$
$= x^2 + 10x + 25$

b) $(y-3)^2$
$= y^2 - 2 \times y \times 3 + 3^2$
$= y^2 - 6y + 9$

c) $(4x-5)^2$
$= (4x)^2 - 2 \times (4x) \times 5 + 5^2$
$= 16x^2 - 40x + 25$

3. Difference of two squares expansion

$(a+b)(a-b)=a(a-b) + b(a-b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

Example: Expand and simplify the difference of two squares

a. $(x+5)(x-5)$ b. $(2x-3)(2x+3)$

Solutions

a) $(x+5)(x-5)$
$= x^2 - 5^2$
$= x^2 - 25$

a) $(2x-3)(2x+3)$
$= 2x^2 - (3)^2$
$= 4x^2 - 9$

Unit: Algebra | **Topic: Simplify Using Index Laws**

Benchmark	**9.3.3.12** Simplify algebraic products and quotients using index laws.

Learning Objectives: By the end of the topic, students will be able to;

- simplify algebraic products using index laws, and
- simplify algebraic quotients using index laws.

Essential questions:

- What is an index law?
- How will I use it to simplify algebraic products and algebraic quotients?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciative and confident. • Critical and participatory.
Skills	• Applying, Reasoning.
Knowledge	• Index Law, algebraic products, algebraic quotients, using index laws to solve the algebraic products and quotients.
Mathematical Thinking	• Apply the understanding of the index laws to simplify algebraic expressions.

Content Background

Index laws

Now that we have these exponential numbers or indices as they are commonly known, we need to be able to perform calculations with them. We need to establish some laws to help us.

Multiplying indices

Consider $3^2 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$ (which is the same as 3^{2+4})

So in general $a^n \times a^n = a^{n+n}$

Dividing indices

Consider $\frac{2^5}{2^3} = \frac{2x2x2x2x2}{2x2x2}$

$= 2 \times 2 = 2^2$ (which is the same as 2^{5-3})

So in general $\frac{a^n}{a^n} = a^{n-n}$

Raising a power to a power

Consider $(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^8$ (which is the same as 3^{2x4})

So in general $(a^n)^n = a^{nxn}$

The Zero Index

Consider $7^4 \div 7^4 = 7^{4-4} = 7^0$, also $7^4 \div 7^4 = \frac{7x7x7x7}{7x7x7x7} = 1$, from this result, we can conclude that any number raised to the power zero is equal to 1.

Negative Index

Consider the table below to find the meaning of $3^{-1}, 3^{-2}, 3^{-3}$

3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
243	$243 x \frac{1}{3} = 81$	$81 x \frac{1}{3} = 27$	$27 x \frac{1}{3} = 9$	$9 x \frac{1}{3} = 3$	$3 x \frac{1}{3} = 1$	$1 x \frac{1}{3} = \frac{1}{3}$	$\frac{1}{3} x \frac{1}{3} = \frac{1}{9}$	$\frac{1}{9} x \frac{1}{3} = \frac{1}{27}$

From the table, we can see that: $3^{-1} = \frac{1}{3}$ $\quad 3^{-2} = \frac{1}{9} = \frac{1}{3^2}$ $\quad 3^{-3} = \frac{1}{27} = \frac{1}{3^3}$

Note: Multiplying a number by $\frac{1}{3}$ is the same as dividing it by 3.

Fractional Indices

Fractional indices can be found using the index laws discussed above.

Example:

a) Use the index laws to simplify$(5^{\frac{1}{2}})^2$.

b) Find $(\sqrt{5})^2$

c) Hence write down the meaning of $5^{\frac{1}{2}}$.

d) Write down the meaning of $24^{\frac{1}{2}}$

e) Write down the meaning of $167^{\frac{1}{3}}$

Solutions

a. $(5^{\frac{1}{2}})^2 = 5^{\frac{1}{2} x 2} = 5^1 = 5$

b. $(\sqrt{5})^2 = 5$

c. Since $(5^{\frac{1}{2}})^2 = (\sqrt{5})^2$ then $5^{\frac{1}{2}} = \sqrt{5}$

d. $24^{\frac{1}{2}} = \sqrt{24}$

e. $167^{\frac{1}{3}} = \sqrt[3]{167}$

Unit: Algebra	Topic: Algebraic Fractions
Benchmark	**9.3.3.13** Apply the four operations to simple algebraic fractions with numerical denominations.

Learning Objectives: By the end of the topic, students will be able to;

- apply addition and subtraction operation with algebraic fractions,
- apply multiplication and division operations with algebraic fractions, and
- apply the four operations with word problems on algebraic fractions.

Essential questions:

- How are four operations applied to solve algebraic fractions?

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate calculation of simple algebraic fractions with confidence and participate with others.
Skills	• Reflect and apply calculation of simple algebraic fractions with the four operations.
Knowledge	• The procession and calculation methods of the four operations for simple algebraic fractions with numerical denominators.
Mathematical Thinking	• Think of formulating other algebraic fractions and apply critically the method of expanding and simplifying the expressions.

Content Background

Algebraic Fractions

Algebraic fractions can be simplified by 'cancelling' the factors which are common in both the numerator and the denominator. We add and subtract multiply and divide algebraic fractions in the same way as we would perform these operations with ordinary fractions.

1. Simplifying Algebraic Fractions

Example: Reduce these algebraic fractions to simplest form

a) $\frac{6x}{12}$ b) $\frac{6a}{7a}$ c) $\frac{8ab}{4a}$ d) $\frac{12pq}{9q}$

Solutions

a. $\frac{6x}{12} = \frac{x}{2}$ b. $\frac{6a}{7a} = \frac{6}{7}$ c. $\frac{8ab}{4a} = 2b$ d. $\frac{12pq}{9q} = \frac{4p}{3}$

2. Addition and subtraction of Algebraic fraction

Example: Simplify

a) $\frac{3x}{10} + \frac{4x}{1}$ b) $\frac{2x}{3} + \frac{x}{4}$ c) $\frac{9m}{10} - \frac{9m}{10}$ d) $\frac{5x}{6} - \frac{3x}{8}$

Solutions

a. $\frac{3x}{10} + \frac{4x}{1}$
$= \frac{3x + 4x}{10}$
$= \frac{7x}{10}$

b. $\frac{2x}{3} + \frac{x}{4}$
$= \frac{4(2x) + 3(x)}{12}$
$= \frac{8x + 3x}{12}$
$= \frac{11x}{12}$

c. $\frac{9m}{10} - \frac{9m}{1}$
$= \frac{9m - 9m}{10}$
$= \frac{3m}{10}$

d. $\frac{5x}{6} - \frac{3x}{8}$
$= \frac{4(5x) - 3(3x)}{24}$
$= \frac{20x - 9x}{24}$
$= \frac{11x}{24}$

3. Multiplication and Division of Algebraic fraction

Example: Simply

a) $\frac{3a}{5} \times \frac{4}{7b}$ b) $\frac{2b}{3c} \times \frac{5d}{7c}$ c) $\frac{5p}{3} \div \frac{10q}{9}$ d) $\frac{4xy}{3} \div \frac{2x}{5}$

Solutions

a. $\frac{3a}{5} \times \frac{8}{7b}$

$= \frac{12a}{35b}$

b. $\frac{2b}{3c} \times \frac{5d}{7c}$

$= \frac{10bd}{21c^2}$

c. $\frac{5p}{3} \div \frac{10q}{9}$

$\frac{5p}{3} \times \frac{9}{10q}$

$= \frac{3p}{2q}$

d. $\frac{4xy}{3} \div \frac{2x}{5}$

$\frac{4xy}{3} \times \frac{5}{2x}$

$= \frac{10y}{3}$

Word problems on Algebraic Fractions

Example 1

$\frac{2}{3}$ of a number is 14. What is the number?

Solution

Step 1: Assign variables

Let x = number

Step 2: Solve the equation

$14 = \frac{2}{3}x$

Isolate variable x

$x = 14 \times \frac{3}{2} = 21$

Answer: the number is 21

Example 2

The numerator of a fraction is 3 less than the denominator. When both the numerator and denominator are increased by 4, the fraction is increased by fraction.

Solution

Let the numerator be x,

Then the denominator x+3

And the fraction $\frac{x}{x+3}$

When the numerator is increased by 4, the fraction is $\frac{x+4}{x+7}$

$\frac{x+4}{x+7} - \frac{x}{x+3} = \frac{12}{77}$

$77(x+4)(x+3) - 77x^2 - 539x = 12(x+7)(x+3)$

$77x^2 + 539x + 924 - 77x^2 - 539x = 12x^2 + 120x + 252$

$12x^2 + 120x - 672 = 0$

$x^2 + 10x - 56 = 0$

$(x-4)(x+14) = 0$

x = 4 (Negative answer not applicable in this case)

Answer: the original fraction is $\frac{4}{7}$

Unit: Algebra	Topic: Substitution
Benchmark	**9.3.3.14** Substitute value into formulae to determine an unknown.

Learning Objectives: By the end of the topic, students will be able to;

- state formulae and determine the unknown, and
- substitute into the subject of the formulae to determine the solutions.

Essential questions:

- How is the substitution method used to determine the unknown values?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be appreciative and confident in solving equations and formulae critically and participate with others.
Skills	• Reflect and use substitution and elimination methods to solve the unknown of the equations.
Knowledge	• Progress with understanding of substitution method, elimination methods in solving formulas and unknown values of equations.
Mathematical Thinking	• Process the substitution and elimination methods critically and evaluate the solutions.

Content Background

Substitution into formulae

A formula is an equation that connects two or more variables. It is normal for one of the variables to be expressed in terms of the other(s). If a formula contains two or more variables and we know the value of all but one of them, we can use the formula to find the value of the unknown variable. Follow the method below;

1. Write down the formulae
2. State the value of the unknown variables
3. Substitute into the formula to find an equation with one variable
4. Solve the equation for an unknown variable

Example:

The area of a triangle is given by $A = \frac{1}{2}bh$ where A is the area, b is the base of the triangle and h is the height.

a) Find the area of the triangle with b = 8 m and h = 7 m

b) Find the height when A = 30 cm^2 and b = 5 cm

Substitute into the formula and solve the unknowns

Solution

a. Through substitution, $A = \frac{1}{2}bh = \frac{1}{2} \times 8 \times 7 = 28m^2$

b. Make h the subject of the formula, $h = \frac{2A}{b} = \frac{2 \times 30}{5} = 12 \quad \therefore h = 12cm$

Strand 4: Statistics and Probability (S & P)

Content Standard: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyze mathematical situations and collect, organize, and represent data to answer questions.

Units	Benchmark	Topics	Lesson Titles
Data and Statistics	**9.4.4.1** Analyse and interpret data using mean, median, mode, range and frequency.	Data	• Types of data (Discrete and Continues Variables) • Calculation of Mean, Mode, Median and range • Analysing & Reporting
	9.4.4.2 Explore data by constructing stem and leaf plots and comparing sets of data.	Exploring data	• Stem and leaf plots • Comparing sets of data
	9.4.4.3 Design a study, collect data, and select the appropriate representation to make conclusions and generalizations.	Data collection, analysis and Representations	• Plan and design data collection method • Data collection • Organization of data • Representation of data
Probability	**9.4.4.4** Compute simple probabilities using appropriate methods such as lists and tree diagrams or through experimental or simulation activities.	Experimental and Theoretical Probabilities	• Introduction to probability • Probability based on experiments • Probability based on theory • Representation of Probability using Tree Diagrams
	9.4.4.5 Identify complementary events and use the sum of probabilities to solve problems.	Complementary events	• Define and identify complementary events • Problem solving using sum of probabilities
	9.4.4.6 Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both).	Language of probability	• Events using 'at least' • Exclusive events • Inclusive events

Unit: Data and Statistics	Topic: Data
Benchmark	**9.4.4.1** Analyse and interpret data using mean, median, mode, range and frequency.

Learning Objectives: By the end of the topic, students will be able to;

- explore and examine types of data (discrete and continuous variables)
- carry out calculation of mean, mode, median and range
- analyse, interpret and report the various types of data

Essential questions:

- What is required to analyse, comprehend and interpret data?
- What is the purpose of collecting, analysing, interpreting and reporting data?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be critical, diligent and rational of data and report meaningfully.
Skills	• Analyse and evaluate types of data and calculate their mean, mode, median and range • Interpret and report various representations of tables and graphs.
Knowledge	• Process of organisation and interpretation of data, mean, median, mode, range and frequency and use of reporting strategies in tables and graph forms.
Mathematical Thinking	• Think of ways and how to organise and analyse data for various purposes and solutions for various audiences in context.

Content Background

1.Data Types

Data is to do with statistics, facts, figures, numbers, records, documents, files etc... Statistics involves the data of information. Data is collected by businesses, government departments and individuals for a wide range of purposes. Population is data, no of boys and girls in the school, students that paid project fees against the number that did not pay are forms of data. Etc.

These data can be represented on tables, charts and graphs of facts and figures for information and a wide range of purposes.
Characteristics or features such as pulse rate, birth-rate, height, age, sex, and eye color are called variables. A quantitative variable is one which measures quantities like height, weight, time etc...

There are 2 types of quantitative variables; Visits to the doctor, the number of road accidents per day and goals in a netball game are all examples of discrete variables. Whereas heights, weight, time measurements are continuous variable.

Data comes in different forms and it needs to be arranged and displayed in a useful form and helpful. It can be tabulated in frequency tables or represented on bar graphs and pie graphs to show that information can be read and understood easily.
Discrete Data is counted and Continues Data is measured

Examples of discrete data

- The number of students in a class
- The results of rolling dice

Examples of continuous data

- A person's height: could be any value within the range of human heights and not fixed
- A dog's weight
- The length of a leaf etc…

2. Calculation of Mean, Mode, median and Range for Ungrouped Data

Example: For the scores in this table, find the

a. Mode b. range c. mean d. median

Score (x)	8	9	10	11	12	13	14	15
Frequency (f)	3	5	12	18	14	5	2	1

Solution

a. The mode is 11 as this is the score with the highest frequency of 18

b. The range is 15 – 8 = 7 (highest score minus lowest score)

c. Add an fx column to find the mean

Score(x)	Frequency(f)	Cummulative Frequency(cf)
8	3	24
9	5	45
10	12	120
11	18	198
12	14	168
13	5	65
14	2	28
15	1	15
	$\sum f=60$	$\sum fx = 663$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{663}{60} = 11.05$$

d. To find the median add a cumulative frequency (cf) column

Score(x)	Frequency(f)	Cummulative Frequency(cf)
8	3	3
9	5	3 + 5 = 8
10	12	8 + 12 = 20
11	18	20 + 18 = 38
12	14	38 + 14 = 52
13	5	52 + 5 = 57
14	2	57 + 2 = 59
15	1	59 + 1 = 60

There are 60 scores so the middle scores are the 30^{th} and 31^{st} scores. These are both 11. The median is 11.

Unit: Data and Statistics	Topic: Exploring Data
Benchmark	**9.4.4.2** Explore data by constructing stem and leaf plots and comparing sets of data.

Learning Objectives: By the end of the topic, students will be able to;

- explore data,
- construct stem and leaf plots, and
- compare sets of data.

Essential questions:

- What are stem and leaf plots and how are they constructed?
- What are the stem and leaf plots used for?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be creative and confident in exploring data and constructing stem and leaf plots and sets of data.
Skills	• Explore and Discover data and ways to construct stem and leaf plots and be able to compare the sets of data.
Knowledge	• Know and understand data and how to explore data and construct stem and leaf plots and sets of data.
Mathematical Thinking	• Analyse data and apply process of exploring data and constructing stem and leaf plots.

Content Background

Investigation:

1. Exploring Data

Data is a collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

Data can be collected using surveys

Collecting Data

Data can be collected in many ways. The simplest way is direct observation.

Example:

- You want to find how many cars pass by a certain point on a road in a 10-minute interval.
- So: stand near that road, and count the cars that pass by in 10 minutes.
- You might want to count many 10-minute intervals at different times during the day, and on different days too!

Data can be represented in many ways

For example using Stem and Leaf Plots

A Stem and Leaf Plot is a special table where each data value is split into a "stem" (the first digit or digits) and a "leaf" (usually the last digit).

Like in this example:

15,16,21,23,23,26,26,30,32,41

Stem	Leaf
1	5 6
2	1 3 3 6 6
3	0 2 this is how you place "32"
4	1

More Examples:

- Stem "1" Leaf "5" means *15*
- Stem "1" Leaf "6" means *16*
- Stem "2" Leaf "1" means *21*
- etc

Example: Long Jump
Sam got his friends to do a long jump and got these results:
2.3, 2.5, 2.5, 2.7, 2.8 3.2, 3.6, 3.6, 4.5, 5.0

And here is the stem-and-leaf plot:

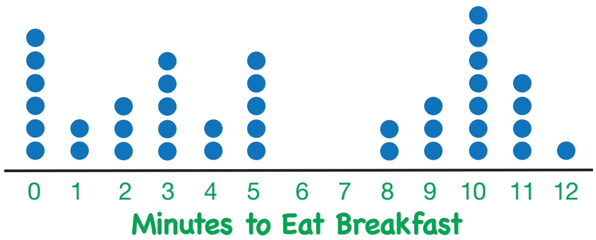
Stem	Leaf
2	3 5 5 7 8
3	2 6 6
4	5
5	0

Stem "2" Leaf "3" means *2.3*

Note:

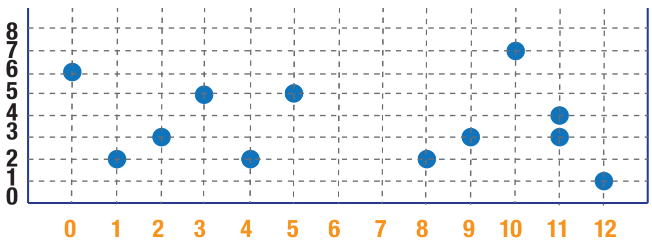
- Say what the stem and leaf mean (Stem "2" Leaf "3" means 2.3)
- In this case each leaf is a decimal
- It is OK to repeat a leaf value
- has a leaf of "0"

A Dot Plot is a graphical display of data using dots
Example: no. of minutes in eating breakfast



Another version of the dot plot has just one dot for each data point like this:

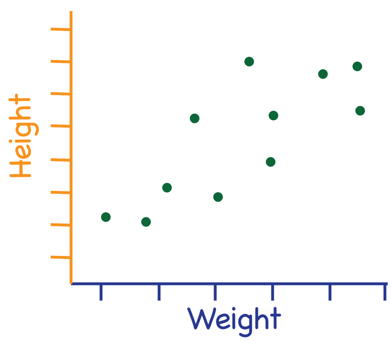
This has the same data as above:
But notice that we need numbers on the side so we can see what the dots mean.



2. Comparing sets of data

For example showing a person's weight and height on a scatter graph

- A Scatter (xy) Plot has points that show the relationship between two sets of data.
- In this example, each dot shows one person's weight versus their height.
- The data is plotted on the graph as Cartesian (x,y) coordinates

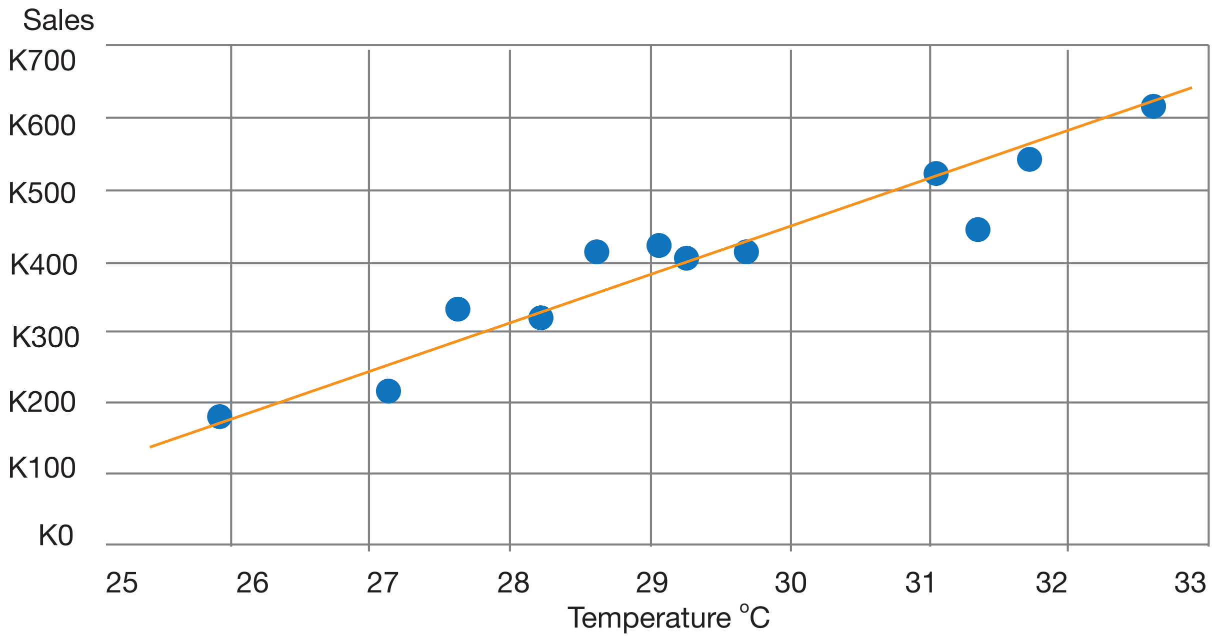


The local ice cream shop keeps track of how much ice cream they sell versus the noon temperature on that day. Here are their figures for the last 12 days:

Ice Cream Sales vs Temperature	
Temperature °C	Ice Cream Sales
25.8°	K180
27.2 °	K205
27.8°	K320
28.5°	K340
28.8°	K402
29.2°	K422
29.4 °	K420
29.8°	K402
31.2°	K544
31.4°	K450
31.8 °	K550
32.5°	K610

It is now easy to see that warmer weather leads to more sales, but the relationship is not perfect.

Line of Best Fit
So we can also draw a "Line of Best Fit" (also called a "Trend Line") on our scatter plot



Try to have the line as close as possible to all points, and as many points above the line as below. But for better accuracy we can calculate the line using Least Squares Regression and the Least Squares Calculator.

Unit: Data and Statistics	Topic: Data Collection, Analysis and Representation
Benchmark	**9.4.4.3** Design a study, collect data, and select the appropriate representation to make conclusions and generalizations

Learning Objectives: By the end of the topic, students will be able to;

- plan and design data collection method,
- carry out data collection,
- organize data,
- represent data.

Essential questions:

- What is required to design data?
- How will the data be organised and represented so that it can be read and interpreted for meaning and purpose?

Key Concepts (ASK-MT)	
Attitudes/Values	• Cooperate with design and collection of data and be responsible in organizing and representing data. • Critically interpret the information and present to audiences.
Skills	• Design, Organise and represent data on various graph and chart representations. • Draw conclusions and explain the information presented.
Knowledge	• Data design, data collection, data organization, data representation
Mathematical Thinking	• Reason and apply process of organising data and representing meaning on graphs and tables etc. and communicate the meaning of the data.

Content Background

1. Plan and design Data
Plan a data collection on the academic performance of grade 9 students in the school. You may decide to collect food consumption per day which could be one of the effects for students' progress and academic performances in the grade.
A class can plan data for 20 students that have no. of meals per day in 10 school days. Select the number of children that you will interview. Draw up questions that the data will be able to answer.

2. Carry out data collection
Use a data collection tool to collect the data by interviewing the students and collecting the data.

- No of students that have 3 meals a day for 10 days
- No of students that have 2 meals a day for 10 days
- No. of students that have 1 meal a day for 10 days

3. Organise Data
Data can be arranged and displayed in a useful form; such as on frequency tables then can be presented on bar graphs and pie-charts so that the information is clear to the audiences or participants or even class mates and etc.

4. Represent data
Then present the data to the parents or teachers, and then they can discuss the findings and their effects on the academic performances of the students. The data should be able to answer your questions such as how many students have only 2 meals a day for the 10 school days.
(can be covered through/as a project for students)

Unit: Probability	Topic: Experimental and Theoretical Probabilities
Benchmark	**9.4.4.4** Compute simple probabilities using appropriate methods such as lists and tree diagrams or through experimental or simulation activities.

Learning Objectives: By the end of the topic, students will be able to;

- define probability,
- compute simple probability based on experiments,
- compute simple probability based on theory, and
- represent simple probability using Tree Diagrams.

Essential questions:

- What is a probability?
- How can we compute simple probability?
- What are simple probabilities based on experiment and theory?
- How can I represent simple probability on a tree diagram?

Key Concepts (ASK-MT)	
Attitudes/Values	• Critically compute probability and represent diligently on tree diagrams.
Skills	• Define probability and compute probability based on experiments and theory • Represent probability using tree diagrams.
Knowledge	• Understanding the process of Experimental and Theoretical Probabilities and how to represent on graphs.
Mathematical Thinking	• Apply ways of computing and representing experimental probabilities.

Content Background

Experimental and Theoretical Probability

Experimental probabilities are obtained through investigating probability events by experiments such as tossing a coin, throwing a die, etc. Thus, the more trials conducted the closer the experimental results are to the theoretical results.

In some instance, it is either very complicated or impossible to calculate the theoretical probability of an event. In this case the relative frequency gives an estimate for the probability.
For example, a cylindrical can is tossed 200 times. The number of times it landed on its side and on an end were recorded in this table.

Outcome	Frequency	Relative Frequency	Percentage
Top End	36	$\frac{36}{200}$	18%
Bottom End	38	$\frac{38}{200}$	19%
Side	126	$\frac{126}{200}$	63%

P(lands on side) = 63%
The definition of probability of an event A occurring is;

$P(A) = \frac{\text{Number of favourable outcomes}}{n}$, where n is the total number in the sample space.

It follows the following probability properties;

- If probability of an event occurring is between 0 and 1
- If P(A) = 0 then the event A is impossible
- If P(A) = 1 then the event A is a certainty
- P(the event A does not occur) = 1-P(A).

Example

A spinner is made from a regular pentagon with equal sections containing the numbers 1, 2, 3, 4, 5.

a. List the sample space
b. Find the probability of spinning a 3
c. Find the probability of spinning an odd number
d. Find the probability of not spinning a 3

Solution

The five numbers 1, 2, 3,4,5 comprise the sample space $\therefore$ n = 5

$P(3) = \frac{1}{5}$

$P(\text{odd number}) = \frac{\textit{Number of odd numbers}}{\textit{number of sample space}} = \frac{3}{5}$

The complementary event to spinning a ‘3’ is ‘not spinning a ‘3’.

$\therefore$ $P(\text{not a } 3) = 1\text{-}P(3)=1\text{-}\frac{1}{5} = \frac{4}{5}$

Unit: Probability	Topic: Complementary Events
Benchmark	**9.4.4.5** Identify complementary events and use the sum of probabilities to solve problems.

Learning Objectives: By the end of the topic, students will be able to;

- define and identify complementary events, and
- solve problem using sum of probabilities.

Essential questions:

- What are complementary events and how will I use the sum of probabilities to solve complementary events?

Key Concepts (ASK-MT)	
Attitudes/Values	• Be creative and responsible for identifying complimentary events and solving problems of probabilities critically and collaboratively with others.
Skills	• Define and identify complementary events and solve problems using sum of probabilities .
Knowledge	• Understand complementary events and sum of probabilities and how to use to solve problems.
Mathematical Thinking	• Reason the occurrences of complimentary events and how to explain using the using sum of probability events.

Content Background

Complementary events

The event in which either event A or B or both occur is called a sum event. When the event A does not occur, this is called a compliment event. The compliment events of probabilities equals to 1.

Example 1. When tossing a coin for head or tail, this is a compliment event as the sum of probability is 1. That is probability for head is 0.5 and probability of a tail is 0.5 which is equal to 1.

P(head)= $\frac{1}{2}$ or P(tail) = $\frac{1}{2}$

The probability of head and tail events complement each other, therefore is a compliment event

Example 2. A tennis racquet is spun 100 times. It has rough on one side and smooth on the other. The table shows the results;

Outcome	Frequency	Relative Frequency	Percentage
Rough	81	$\frac{81}{100}$	81%
Smooth	19	$\frac{19}{100}$	19%

P (Rough) = $\frac{81}{100}$ or

P(Smooth) = $\frac{19}{100}$; the compliment of this probability is equals to 1, compliment event of rough is smooth and the compliment of smooth is rough which always add to 1.

Unit: Probability	Topic: Language of Probability
Benchmark	**9.4.4.6** Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and and’for probability events.

Learning Objectives: By the end of the topic, students will be able to;

- describe events using ‘at least’,
- describe events using ‘exclusive events’,
- describe events using ‘inclusive events’.

Essential questions:

- What language is used to describe the probability events?

Key Concepts	
Attitudes/Values	• Be creative and responsible in the use of the probability language to describe various events critically and collaboratively with others.
Skills	• Describe events using the language of ‘at least’, ‘exclusive events’ and ‘inclusive events’.
Knowledge	• Understand and how to use language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and and’ for probability events.
Mathematical Thinking	• Use language reasonably and communicate the events to the audiences.

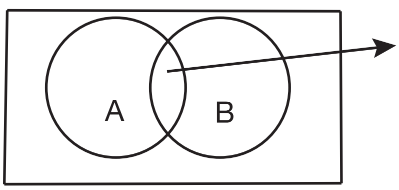
Content Background

The Language of probability
Probability is a measurement of the chance or likelihood of an event happening.
The words that we might use to describe probabilities include;

- Unlikely,
- 50-50 chance,
- Likely,
- Poor chance,
- Possible,
- Certain,
- Very likely,
- Impossible,
- Probable,
- Even chance.

Multiplication Theorem
If the question has “both” or “A and B” or “all”, this means intersection of the sets; use multiplication theorem. If the question has ‘A given B’ it means conditional probability.

- Independent events: Probability of A & B, P(AB) = P(A)*P(B)
- If not dependent: P(AB) = P(A)*P(B/A)
- Conditional Probability: $P(A/B) = \frac{P(A \cap B)}{P(B)}$



Probability of A = P(A)
Probability of B = P(B)
Probability of A given B = P(A/B)

‘At least A’ or B, A and B, A or B exclusive, A or B inclusive

1. Events using 'at least';

If probability of two events occurring at the same time or together. Independently use the specific multiplication rule formula. Just multiply the probability of the first event by the second.
For example if the probability of event A is 2/9 and the probability of event B is 3/9 then the probability of both events is $\frac{2}{9} \times \frac{3}{9} = \frac{6}{81} = \frac{2}{27}$

2. Inclusive events

Inclusive events are events that can happen at the same time. To find the probability of an inclusive event; we first add the probabilities of the individual events and then subtract the probability of the two events happening at the same time.

P(9 or less than 3) – find the probability of rolling a 9 or a number less than 3
Step 1: find the probability of each event independently
P(9) = 1/10 There's 1 nine on the die
There are 10 outcomes on the die
P(less than 3) = 2/10 two numbers less than 3 (1 & 2)
10 outcomes on the die

Step 2: add the probability of each individual event
P(9 or less than 3) = $\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
The probability of rolling a 9 or a number less than 3 on a 10 sided die is 3/10

3. Exclusive events

Two events are mutually exclusive when they cannot happen at the same time. The probability that one of the mutually exclusive events occur is the sum of their individual probabilities. An example of two mutually exclusive events is a wheel of fortune.

Let's say you may win a bar of chocolate but may end up in a red or a pink field.

A – you are at school
B – You are bowling
$A \cap B$ – You are at school and you are bowling
$A \cap B = \varnothing$ (you can't bowl at school)

Standards-Based Lesson Planning

What are Standards-Based Lessons?

In a Standards-Based Lesson, the most important or key distinction is that, a student is expected to meet a defined standard for proficiency. When planning a lesson, the teacher ensures that the content and the methods of teaching the content enable students to learn both the skills and the concepts defined in the standard for that grade level and to demonstrate evidence of their learning.

Planning lessons that are built on standards and creating aligned assessments that measure student progress towards standards is the first step teacher must take to help their students reach success. A lesson plan is a step-by-step guide that provides a structure for an essential learning.

When panning a standards-based lesson, teacher instructions are very crucial for your lessons. How teachers instruct the students is what really points out an innovative teacher to an ordinary teacher. Teacher must engage and prepare motivating instructional activities that will provide the students with opportunities to demonstrate the benchmarks. For instance, teacher should at least identify 3-5 teaching strategies in a lesson; teacher lectures, ask questions, put students into groups for discussion and role play what was discussed.

Why is Standards-Based Lesson Planning Important?

There are many important benefits of having a clear and organized set of lesson plans. Good planning allows for more effective teaching and learning. The lesson plan is a guide and map for organizing the materials and the teacher for the purpose of helping the students achieve the standards. Lesson plans also provide a record that allows good, reflective teachers to go back, analyse their own teaching (what went well, what didn't), and then improve on it in the future.

Standards-based lesson planning is vital because the content standards and benchmarks must be comparable, rigorous, measurable and of course evidence based and be applicable in real life that we expect students to achieve. Therefore, teachers must plan effective lessons to teach students to meet these standards. As schools implement new standards, there will be much more evidence that teachers will use to support student learning to help them reach the highest levels of cognitive complexity. That is, students will be developing high-level cognitive skills.

Components of a Standards-Based Lesson Plan

An effective lesson plan has three basic components;

- aims and objectives of the course,
- teaching and learning activities,
- assessments to check student understanding of the topic.

Effective teaching demonstrates deep subject knowledge, including key concepts, current and relevant research, methodologies, tools and techniques, and meaningful applications.

Planning for under-achievers NORMA

Who are underachieving students?
Under achievers are students who fail or do not perform as expected. Underachievement may be caused by emotions (low self-esteem) and the environment (cultural influences, unsupportive family)

How can we help underachievement?
Underachievement varies between students. Not all students are in the same category of underachievement.

Given below a suggested strategies teachers may adopt to assist underachievers in the classroom.

- Examine the Problem Individually
  It is important that underachieving students are addressed individually by focusing on the student's strengths.

- Create a Teacher-Parent Collaboration
  Teachers and parents need to work together and pool their information and experience regarding the child. Teachers and parents begin by asking questions such as;

  - In what areas has the child shown exceptional ability?
  - What are the child's preferred learning styles?
  - What insights do parents and teachers have about the child's strengths and problem areas?

- Help student to plan every activity in the classroom
- Help students set realistic expectations
- Encourage and promote the student's interests and passions.
- Help children set short and long-term academic goals
- Talk with them about possible goals.
- Ensure that all students are challenged (but not frustrated) by classroom activities
- Always reinforce students

Example of Standards-Based Assessment Planning

The following sample lesson can help teachers to plan effective lessons. Teachers are encouraged to study the layout of the different components of these lessons and follow this design in their preparation and teaching of each lesson. Planning a good lesson helps the teacher in maintaining a standard teaching pattern which should not deviate students learning of the concept from the topic.

SAMPLE STANDARDS-BASED LESSON PLAN

TOPIC: Equation of a straight line

Content Standard 3: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyse mathematical situations and collect, organise, and represent data to answer questions.

Unit 1: Linear Functions **Topic: Equation of a straight line**

Benchmark: 9.3.3.1 Determine the slope and equation of a line when given the graph of a line, two points on the line, or the equation of the line.

Lesson Title: Gradient of a straight line ($y = mx+c$)

Learning Objectives: By the end of this topic, students will be able to;

- identify and sketch a straight line with the corresponding pairs of (x,y) values and define graph of linear function ($y = mx+c$)
- define and conclude the equation of a straight line with y-intercept and gradient
- define and conclude the equation given two points, one point and the gradient and given the graph

Materials: Content and activity handouts

Essential Knowledge, Skills, Values, and Attitudes

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate the construction of straight line graphs and critically evaluate the equations of the straight line graphs.
Skills	• Construct straight line graphs and determine their equations line. • Evaluate and draw conclusions to the equation of a straight line.
Knowledge	• Gain understanding of corresponding x, y values, linear function $y = mx + c$, $m \neq 0$, gradient = rise over run (difference of the coordinates or points on the line), y intercept, straight line equation, $y = ax + b$ • Assimilate the equations of the straight line graphs as a linear function.
Mathematical Thinking	• Think about, reason and communicate the conclusions to the equations of the straight line using gradient, corresponding points and y intercept.

Lesson Procedure

Teacher Activities	Student Activities
Introduction	**(5mins)**
• Recap students' previous knowledge on straight line equation and graphs. • Introduce straight line equation y= 4x+1 and discuss its properties and stress and its essential points.	• Use their previous knowledge and share their ideas on the teachers' respective questions. • Think about the equation of the straight line y=4x+1 and discuss with teacher on its essential properties.
Body	**(20 mins)**
Modelling	
• Model through using the straight line equation y= 4x+1 by asking students to find the values of y when x = 0,1,2,3 etc • Ask students to use the values of x,y to plot the graph of y=4x+1 on the board.	• Think about how to find respective values of y when x is given. • Discuss with the teacher to plot the graph of y=4x+1 using the x,y values found above.
Guided Practice	
• Ask students to determine the gradient the gradient and y-intercept of the graph y= 4x+1 with and without using the graph. • Allow for discussion on how the gradient and y – intercept can be found with and without using the graph.	• Think about how to find the gradient and y-intercept of y= 4x+1 using the graph and without using the graph. • Discuss with the teacher to find the gradient and y-intercept of the straight line with and without using the graph.
Independent Practice	
• Provide 2 straight line equations and ask students to find the gradient and y-intercepts for each of them.	• Individual students think about how to find the gradient and the y-intercepts of the two respective straight line equations.
Conclusion	**(15 mins)**
• Allow students to present their solution on the board for class discussion. • Make corrections where necessary on the student's presentations and stress on the key points.	• Present and write their solutions on the board and explain their answers. • Make necessary corrections if any through consolidating the key points of the lesson highlighted by the teacher.

Assessment/Lesson Evaluation
Students can; • identify and sketch a straight line with the corresponding pairs of (x,y) values and define graph of linear function (y=mx +c) • define and conclude the equation of a straight line with y-intercept and gradient.

Assessment, Monitoring and Reporting

What is Standards-Based Assessment (SBA)?

Standards-Based Assessment is an on-going and a systematic process of **assessing, evaluating, reporting** and **monitoring** students' performance and progression towards meeting grade and national level expectations. It is the measurement of students' proficiency on a learning objective or a specific component of a content standard and progression towards the attainment of a benchmark and content standard.

Step.1: Identify what (knowing/ skills) to assess .

Step.2: Clearly describe the purpose of assessment(what is that you intend to do by assessing students' peformance

Step.3: Prepare the assessment task to assess students' performance

Step.4: Devise a scoring method for scoring students' performance (performnace

Step.5: Identify the expected level of proficiency

Step.6: Develop a descriptor for each performnce standard or criteria

Step.7: Identify qualifiers for performance lebels (Qualifiers are used to define each of the levels of performance (e.g. all for proficient, few for progressing, one for not yet or Excellent

Purpose of Standards-Based Assessment

Standards-Based Assessment (SBA) serves different purposes. These include instruction and learning purposes. The primary purpose of SBA is to improve student learning so that all students can attain the expected level of proficiency or quality of learning.

Enabling purposes of SBA is to:

- measure students' proficiency on well-defined content standards, benchmarks and learning objectives
- ascertain students' attainment or progress towards the attainment of specific component of a content standard
- ascertain what each student knows and can do and what each student needs to learn to reach the expected level of proficiency
- enable teachers to make informed decisions and plans about how and what they would do to assist weak students to make adequate progress towards meeting the expected level of proficiency
- enable students to know what they can do and help them to develop and implement strategies to improve their learning and proficiency level
- communicate to parents, guardians, and relevant stakeholders the performance and progress towards the attainment of content standards or its components
- compare students' performances and the performances of other students

Principles of Standards-Based Assessment

The principle of SBA is for assessment to be;

- emphasing on tasks that should encourage deeper learning
- be an integral component of a course, unit or topic and not something to add on afterward
- a good assessment requires clarity of purpose, goals, standards and criteria
- of practices that should use a range of measures allowing students to demonstrate what they know and can do
- based on an understanding of how students learn
- of practices that promote deeper understanding of learning processes by developing students' capacity for self-assessment
- improving performance that involves feedback and reflection
- on-going rather than episodic
- given the required attention to outcomes and processes
- be closely aligned and linked to learning objectives, benchmarks and content standards

Standards-Based Assessment Types

In standards-Based Assessment, there are three broad assessments types.

1. Formative Assessment

Formative assessment includes 'assessment *for* and *as'* and is conducted during the teaching and learning of activities of a topic.

Purposes of assessment for Learning

- On-going assessment that allows teachers to monitor students on a day-to-day basis.
- Provide continuous feedback and evidence to the teachers that should enable them to identify gaps and issues with their teaching, and improve their classroom teaching practice.
- Helps students to continuously evaluate, reflect on, and improve their learning

Purposes of assessment as Learning

- Occurs when students reflect on and monitor their progress to inform their future learning goals
- Helps students to continuously evaluate, reflect, and improve their own learning
- Helps students to understand the purpose of their learning and clarify learning goals

2. Summative Assessment

Summative assessment focuses on 'assessment *of learning'* and is conducted after or at the conclusion of teaching and learning of activities or a topic.

Purposes of assessment of Learning

- Help teachers to determine what each student has achieved and how much progress he/she has made towards meeting national and grade-level expectations
- Help teachers to determine what each student has achieved at the end of a learning sequence or a unit.
- Enable teachers to ascertain each student's development against the unit or topic objectives and to set future directions for learning.
- Help students to evaluate, reflect on, and prepare for next stage of learning

3. Authentic Assessment

- Is performed in a real life context that approximates as much as possible, the use of a skill or concept in the real world.
- Is based on the development of a meaningful product, performance or process
- Students develop and demonstrate the application of their knowledge, skills, values and attitudes in real life situations which promote and support the development of deeper levels of understanding.

Authentic assessment refers to assessment that:

- Looks at students actively engaged in completing a task that represents the achievement of a learning objective or standard
- Takes place in real life situations
- Asks students to apply their knowledge, skills, values and attitudes in real life situations
- Students are given the criteria against which they are being assessed

Performance Assessment
Performance assessment is a form of testing that requires students to perform a task rather than select an answer from a ready-made list. For example, a student may be asked to explain historical events, generate scientific hypotheses, solve math problems, converse in a foreign language, or conduct research on an assigned topic. Teachers, then judge the quality of the student's work based on an agreed-upon set of criteria. It is an assessment which requires students to demonstrate that they have mastered specific skills and competencies by performing or producing something.

Types of performance assessment;

i. Products
This refers to concrete tangible items that students create through either the visual, written or auditory media such as;

- Creating a health/physical activity poster
- Video a class game or performance and write a broadcast commentary
- Write a speech to be given at a school council meeting advocating for increased time for health and physical education in the curriculum
- Write the skill cues for a series of skill photo's
- Create a brochure to be handed out to parents during education week

- Develop an interview for a favourite sportsperson
- Write a review of a dance performance
- Essays
- Projects

ii. Process Focused Tasks
It shows the thinking processes and learning strategies students use as they work such as;

- Survival scenarios
- Problem solving initiative/adventure/ activities
- Decision making such as scenario's related to health issues
- Event tasks such as creating a game, choreographing a dance/gymnastics routine, creating an obstacle course
- Game play analysis
- Peer assessment of skills or performances
- Self-assessment activities
- Goal setting, deciding a strategy and monitoring progress towards achievement

iii. Portfolio
This refers to a collection of student work and additional information gathered over a period of time that demonstrates learning progress.

iv. Performances
It deals with observable affective or psycho-motor behaviours put into action such as;

- Skills check during game play
- Role plays
- Officiating a game
- Debates
- Performing dance/gymnastics routines
- Teaching a skill/game/dance to peers

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the "be able to do" of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They include explanations of how well students must demonstrate the content, explaining how good is good enough.

Performance standards:

- measure students' performance and proficiency (using performance indicators) in the use of a specific knowledge, skill, value, or attitude in real life or related situations
- provide the basis (performance indicators) for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude
- are used to plan for individual instruction to help students not yet meeting expectations (desired level of mastery and proficiency) to make adequate progress towards the full attainment of benchmarks and content standards
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards

Assessment Strategies

It is important for teachers to know that, assessment is administered in different ways. Assessment does not mean a test only. There are many different ways to find out about student's strengths and weaknesses. Relying on only one method of assessing will not reflect student's achievement.

Provided in the appendices is a list of suggested strategies you can use to assess student's performances. These strategies are applicable in all the standards-based assessment types.

How to Develop effective SBA tools

Teachers are required to use the steps outlined below when planning assessment. These steps will guide you to develop effective assessments to improve student's learning as well as evaluating their progress towards meeting national and grade –level expectations.

Step.1: Identify what (knowing/ skills) to assess .

→ **Step.2**: Clearly describe the purpose of assessment(what is that you intend to do by assessing students' perormance

→ **Step.3:** Prepare the assessment task to assess students' performance

→ **Step.4**: Devise a scoring method for scoring students' performance (performnace

→ **Step.5**: Identify the expected level of proficiency

→ **Step.6**: Develop a descriptor for each performnce standard or criteria

→ **Step.7**: Identify qualifiers for performance lebels (Qualifiers are used to define each of the levels of performance (e.g. all for proficient, few for progressing, one for not yet or Excellent

Samples of Assessment Types

Formative

Sample 1

STRAND 3: Patterns and Algebra

Content Standard 3: Students will be able to interpret various types of patterns and functional relationship, use symbols forms to represent, model, and analyze mathematical situations, and collect, organize, and represent data to answer questions.

Benchmarks: 9.3.3.1 Determine the slope and equation of a line when given the graph of a line, two points on the line, or the equation of the line

Unit 1: Linear Functions

Topic: Equation of a straight line

Lesson title: Finding equation of a straight line given gradient and y – intercept.

Objective 1: By the end of the lesson, students should be able to:
evaluate the x;y values from the corresponding pairs of (x,y) values of quantities
find the y intercept and gradient from y = mx + c
formulate an equation of the straight line

Materials: Activity handouts and graph papers

What is to be assessed? (ASK-MT)

Key Concepts (ASK-MT)	
Attitudes/Values	• Appreciate the construction of straight line graphs and critically evaluate the equations of the straight line graphs.
Skills	• Construct straight line graphs and determine their equations line. • Evaluate and draw conclusions to the equation of a straight line.
Knowledge	• Gain understanding of corresponding x, y values, linear function y = mx + c, m≠0, gradient = rise over run (difference of the coordinates or points on the line), y intercept, straight line equation, y = ax + b • Assimilate the equations of the straight line graphs as a linear function.
Mathematical Thinking	• Think about, reason and communicate the conclusions to the equations of the straight line using gradient, corresponding points and y intercept.

Purpose of the assessment

To measure students' proficiency on the achievement of the benchmark and learning objectives.

Expected level of proficiency:

- Evaluation of the given corresponding pairs of (x,y) values
- Calculation of the gradient or slope of straight line graph
- Finding the y intercept when x = 0
- Formulation of the equation of the straight line

Performance Task

Students will be given an activity for them to work in pairs or individuals to formulate an equation from the table of corresponding x,y values.

Assessment Strategy

This activity can be assessed in one lesson to determine how well students can understand how to formulate equations for straight line graphs given the gradient and y-intercept.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for a lesson exercise.

Key Concepts and Skills					
Performance Standards/ Criteria	A	B	C	D	Score
	Advance 10	**Proficient 9-5**	**Progressing 3-4**	**Not Yet 2**	**__/10 Marks**
Formulate an equation of the straight line given gradient and y–intercept.	• Evaluate the given corresponding pairs of (x,y) values • Calculate the gradient or slope of straight line graph • Find the y-intercept when x = 0 • Formulate the equation of the straight line • Share their ideas with others.	• Evaluate the given corresponding pairs of (x,y) values • Calculate the gradient or slope of straight line graph • Find the y-intercept when x = 0 • Formulate the equation of the straight line	• Evaluate the given corresponding pairs of (x,y) values • Calculate the gradient or slope of straight line graph	• Difficult to Evaluate the given corresponding pairs of (x,y) values	

Summative
Sample 2

STRAND 2: GEOMETRY, MEASUREMENT AND TRANSFORMATION

Content Standard 2: Students will be able to comprehend the meaning and significance of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulae for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyze mathematical situations.

Unit: RATES
Benchmark 9.2.2.6- 9.2.2.8:
(refer to the benchmarks in unit: Rates of Strand 2)

Topics:
(refer to the topics in unit: Rates of Strand 2)

Lesson topics:
(refer to the lesson topics in unit: Rates of Strand 2)

Instructional Objective(s):
(refer to unit: Rates of Strand 2)

What is to be assessed? - (ASK-MT)

VASK-MT	
Values/ Attitudes	• Show confidence in solving problems on rates.
Skills	• Solve problems on rates.
Knowledge	• Rates
Scientific Thinking	• Think about the processes involved to solve rates problem.

Purpose of the assessment

To measure students' proficiency on the achievement of the benchmarks and learning objectives in this unit. (This assessment is to be conducted after teaching the unit)

Expected level of proficiency

All students are expected to;

- investigate very small and very large time scales and intervals.
- explain representation of tables and graphs of rates.
- solve problems with rates and interpret related graphs.

Performance Task

Students will do an assignment out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.
Task: Students are to:

- Do an assignment out of 20 marks on the Rates covering benchmarks 9.2.2.6 to 9.2.2.8
- Show all the calculation to attain full marks.

Assessment Strategy

An assignment will be used to measure students' proficiency.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for an assignment.

Key Concepts and Skills					
Performance Standards/ Criteria	Model/ Exemplar	Proficient	Developing	Beginning	Score
	(20 points)	(13-19 points)	(6-12 points)	(2-5 points)	_
Quality/ Effort	Maximum effort was put forth to complete the assignment in a professional manner. Assignment demonstrates a high degree of quality and attention to detail.	Some effort was made to complete the assignment to a level that was sufficient for grading, but does not meet a professional level of quality or appearance.	Minimal effort was made to complete the assignment but still meets the minimal standard.	Little or no effort was made to produce a quality assignment. Assignment obviously does not meet minimal standards.	
Mathematical Calculations	All calculations are very clear, organized, and neatly completed with no inaccuracies.	All calculations are clear, organized, and neatly completed with 1-2 inaccuracies.	Most calculations are clear, organized, and neatly completed with 3-4 inaccuracies.	Calculations are unclear and disorganized and 5 or more inaccuracies may be present.	

Authentic Assessment

Sample 3

Strand 4: STATISTICS AND PROBABILITY

Content Standard 4: Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

Unit : Data and Statistics

Benchmark 9.4.4.1 – 9.4.4.4 (refer to the benchmarks in unit: Data and Statistics Strand 4)

Topics: (refer to the topics in unit: Data and Statistics, Strand 4)

Lesson topics: (refer to the topics in unit: Data and Statistics, Strand 4)

Instructional Objective (s): (refer to the topics in unit: Data and Statistics, Strand 4)

What is to be assessed? - (ASK-MT)

Key Concepts (ASK-MT)	
Attitudes/Values	• Cooperate with design and collection of data and be responsible in organizing and representing data.
Skills	• Design, Organise and represent data on various graph and chart representations and draw conclusions and explain the information presented.
Knowledge	• Data design, data collection, data organization, data representation
Mathematical Thinking	• Reason and apply process of organising data and representing meaning on graphs and tables etc. communicate the meaning of the data.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives in this unit. This assessment is to be conducted after teaching this unit.

Expected level of proficiency

All students are expected to;

- Design a study, collect data, and select the appropriate representation to make conclusions and generalizations.

Performance Task

Students will do a project out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.

Task

Your task is to plan a data collection on the academic performance of grade 9 students in the school. You may decide to collect food consumption per day which could be one of the effects for students' progress and academic performances in the grade.

Students will be given two week to complete this project.

Task Details

- Plan data for 20 students that have no of meals per day in 10 school days. Select the number of children that you will interview.
- Use appropriate data collection tool to collect the data by interviewing the students and collecting the data.
- Data must be arranged and displayed in a useful form; such as on frequency tables then can be presented on bar graphs and pie- charts so that the information is clear to the audiences.
- Presentation of your finding will be done to the parents or teachers on the effects of meals on the academic performances of the students.

Assessment Strategy

A project will be used to measure students' proficiency.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for the project.

Category	Advanced	Satisfactory	Partial Credit	Unacceptable
	9-10 points	7-8 points	1-6 points	0 points
Quality	Maximum effort was put forth to complete the project in a professional manner. Project demonstrates a high degree of quality and attention to detail. Workmanship is excellent.	Some effort was made to complete the project to a level that was sufficient for grading, but does not meet a professional level of quality or appearance. Workmanship is of acceptable quality.	Minimal effort was made to complete the project and the quality and workmanship is sub-par, but still meets the minimal standard.	Little or no effort was made to produce a quality project.
Data Collection	Evidence of data collection very clear, organized, and neatly completed with no inaccuracies.	Evidence of data collection clear, organized, and neatly completed with few inaccuracies.	Evidence of data collection not very clear, organized, and completed with some inaccuracies.	No evidence of data collection, not organized, and not completed

Organising Data and Representing	Data represented/ displayed using frequency tables, bar graphs, pie charts etc... Information appropriately scaled and labeled.	Data represented/ displayed using frequency tables, bar graphs, pie charts etc... Some information not appropriately scaled and labeled.	Only few data represented.. Information not clear and not appropriately scaled and labeled.	No attempt done on organizing data.
Time Management	Project completed and turned in on time. Student worked diligently when project time was available. Student was on task most of the time.	Project was completed, but had notable errors. Student utilized project time somewhat efficiently, but spent time socializing. Student was on task 70% - 80% of the time.	Project was not turned in on time and/or complete. The student was on task less than 60% of the time.	Project was not turned in on time and was not completed. Student wasted project time and at times was disruptive to others.
Team Work	Notable teamwork shown with a determination to participate/ contribute to team success. Completed required individual tasks that contributed to the success of the team	Teamwork was noted, but was sometimes off task or working on non-related tasks. Contributed to the success of the team, but could have been more engaged to complete tasks sooner.	Notable time off-task with minimal effort given for team success, or did the project alone without relying on others to do their share of the project.	Was not a team player. Either took over project completely, or did not engage in team direction or plans.
Pressentation	Presentation was well organized and presented in a logical sequence. Presentation reflects a full knowledge of the topic with clear answers and explanations to questions asked.	Presentation was fairly organized and most information presented in a logical sequence. Answers to questions were vague or lacked clarity or accuracy.	Presentation was unorganized and lacked a logical sequence. Presentation reflected little attention to detail. Answers to questions were inaccurate and confusing.	Presentation was not acceptable and reflects a lack of organization or knowledge of the topic. Presentation shows little effort to meet expectations.

STEAM Assessment

Sample 4; (Integrated Strands in relation to the project from integrated subjects)

Unit: (Integrated Units from all Subjects in this project)

Content Standard: (Integrated Content Standard from all Subjects in project)

Benchmark: (Integrated Benchmarks from all Subjects in this project)

Topic: (Integrated Topics from all Subjects in this project)

Lesson topic: (Integrated Topics from all Subjects in concern)

Instructional Objective (s): Students will be able to;

- Create a STEAM project "building a prototype model of a catapult launching system" to enhance their understand of this concept

What is to be assessed? (KSAVs

VASK-MT	
Values/Attitudes	Appreciate the beauty of the application of mathematics during the designing process of the project.
Skills	Calculating size and space Time management and efficiency, Linear measurement and scaling techniques, Calculating mechanical advantage
Knowledge	Size and space Time management and efficiency, Linear measurement and scaling techniques
Mathematical Thinking	Think about how to integrate and apply the mathematical knowledge in the project

Integrated subjects concepts used designing the projects.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives for integrated subjects in the project. (STEAM Project)

Expected level of proficiency

All students are expected to;

- "Build a prototype model of a catapult launching system" through integrating concepts learned in other subjects.

Performance Task

Student will carry out a project worth 30 marks that should contribute to the School Learning Improvement Program (SLIP). This project will assess students proficiency on the mentioned benchmarks. In order for this assessment type to attain its intended purpose the following must be done carefully;

Task: Students will be given a month to complete this project.

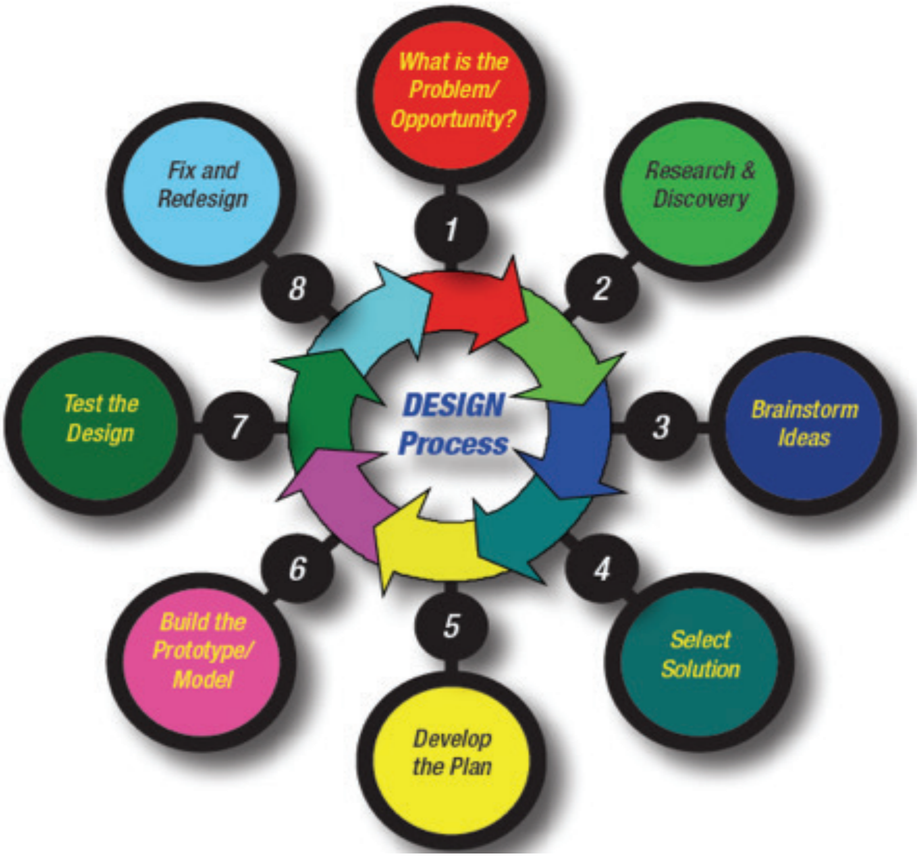
(1) all grade 9 Mathematics teachers discuss the STEAM project with their HOD
(2) the Mathematics HOD brings this project to the attention of the Head Teacher hence it will involve the learning of all grade 9 classes in the school.
(3) once approved by the Head Teacher, the Mathematics HOD now convenes a meeting with all other subject HOD to integrate this project into their learning. HOD for Mathematics will have developed criteria already and will discuss around that.
(4) the HOD for other subjects meet with their respective subject teachers to gauge their views and write up criteria's with reference to the theme of the project, "STEM Design and Engineering Challenge" bringing out the essence of their subjects in this project.
(5) the Head Teacher then convenes a meeting with all teachers as they are now aware of the project. HOD for respective subjects give feedback from their meetings. Issues concerning this project must be ironed out and all subjects now carry out this assessment, starting with Mathematics.

The grade 9 Mathematics teacher will now do the following;

(i) Group the students into groups of 6 to design (drawing and manual) a tangible technology that will enhance the notion of "building a prototype model of a catapult launching system"
(ii) The teacher then assesses their designs and the best designs now compete with the other best designs from other grade 9 classes.
(iii) All the best designers now create models of their designs with assistance from their class members. At this stage the other subjects now carry forward this assessed projects theme, 'building a prototype model of a catapult launching system" however in the context of their subjects. STEAM is an integrated approach of teaching. All subjects must incorporate the theme put forward by Mathematics. They develop criteria that should address this theme. For instance; Technology and Industrial Arts (TIA) will develop criteria that will engage the students to construct the models. Science teachers will develop criteria to test students' knowledge of the Science process of Engineering Design thinking when they create the models around the theme of "prototype model of a catapult launching system". The English subject teachers will set criteria and guidelines for students on how to write reports so they write to tell others what they have learned and experienced. They must also be given guidelines to writing report. Students get to write report of how they designed this technology. The Mathematics teacher will provide criteria for the students in terms of the measurements, angles and operations used to work out the size and shape of the technology.

Task: Students will be given 6 weeks to complete this project. They are to;

- Design and build a prototype model of a catapult launching system that is easy to use and easy to transport.
- Follow the Design Process to prepare their prototype model in time.
- Write and prepare a short presentation to explain the catapult that was built and the process of building it.



Design Specification:

The catapult should be designed to launch a golf ball at least fifteen feet, to a 18cm x 18cm target.

- The catapult should include a system for determining range, reliability, and accuracy.
- The catapult should be mobile, yet stable. Outriggers or other support systems need to be included to maintain stability when the launcher is used.
- The catapult should be no larger than 30cm long x 30 cm deep x 90cm tall.
- The catapult should feature a locking pin or trigger that activates the catapult to launch.
- Your team should prepare to deliver a presentation about the merits of your catapult model and design.

Assessment Strategy

Design Project will be used to measure student's proficiency.

The students will be reinforced in the following STEAM concepts.

Science

- Applications of simple machines, including wheels and axles, levers, and pulleys
- Balance and equilibrium
- Energy transformations, such as rotary motion to linear motion
- Mechanical advantage

Technology and Engineering

- Prototyping and modelling
- Invention and innovation
- Structural integrity/strength
- Brainstorming and problem solving
- Trial and error engineering concepts

ARTS

- Perspective drawing (3-D)
- Critical Thinking Process
- Applying the Principles of Graphic design
  - balance
  - proximity
  - repetition
  - colour
  - negative/positive space
- Applying creative process

Math

- Calculating size and space
- Time management and efficiency
- Linear measurement and scaling techniques
- Calculating mechanical advantage

Project Rubric

Category	Advanced	Satisfactory	Partial Credit	Unacceptable
	9 -10 points	7- 8 points	1 - 6 points	0 points
Quality/ Workmanship	Maximum effort was put forth to complete the project in a professional manner. Project demonstrates a high degree of quality and attention to detail. Workmanship is excellent.	Some effort was made to complete the project to a level that was sufficient for grading, but does not meet a professional level of quality or appearance. Workmanship is of acceptable quality.	Minimal effort was made to complete the project and the quality and workmanship is sub-par, but still meets the minimal standard.	Little or no effort was made to produce a quality project. Project obviously does not meet minimal standards.
Creativity/ Design	Project reflects many fundamental elements of design and creativity. Project demonstrates an advanced understanding of creative thinking and attention to aesthetics and presentation.	Project reflects some of the elements of design and creativity, but lacks attention to aesthetics and presentation.	Project was completed, but does not reflect the acceptable levels of design and creativity. Effort was minimal and project is mediocre at best.	Project was not completed on time or reflects little or no effort to complete assignment at an acceptable level.
Functionality	Project meets or exceeds the design requirements of purpose and functionality. All elements of the design have been met and the project does what it was designed to do.	Project meets some of the design requirements of purpose and functionality. Not all elements of the design have been met, but the project does what it was designed to do.	Project is somewhat functional, but reflects minimal effort. It is intermittent and doesn't always do what it was designed to do.	Project does not work and demonstrates a lack of effort or understanding of the basic elements of functionality and purpose.
Design Process	Project reflects a clear understanding and application of design process including evidence of research, brainstorming, design and problem solving, prototyping and testing.	Project reflects some understanding and application of accepted design loop principles and sequence including evidence of research, brainstorming, design and problem solving, prototyping and testing.	Project reflects minimal understanding and application of design process.	Project does not show evidence that design process was used. Project does not meet accepted levels of design criteria.
Criteria/ Constraints	Project was completed with all constraints and criteria met or exceeded. Reflects attention to detail and quality.	Project was completed with some of the constraints and criteria met. Reflects some attention to detail, but quality is minimal.	Project was completed with a few of the constraints and criteria met. Reflects minimal effort and lacks detail or quality.	Project was not completed and does not reflect the adherence to the constraints or criteria.

Time Management	Project completed and turned in on time. Student worked diligently when project time was available. Student was on task most of the time.	Project was completed, but had notable errors. Student utilized project time somewhat efficiently, but spent time socializing. Student was on task 70% - 80% of the time.	Project was not turned in on time and/or complete. The student was on task less than 60% of the time.	Project was not turned in on time and was not completed. Student wasted project time and at times was disruptive to others.
Resource Management	Always takes responsibility for use and care of all building components and resources. Always returns building components and materials to proper storage compartments.	Consistently takes responsibility for use and care of building components and resources. Somewhat consistent in returning building components to proper storage compartments.	Sometimes takes responsibility for use and care of building components and resources. Inconsistent in returning building components to proper storage compartments.	Does not take responsibility for the proper use and care of building components and resources. Is careless and does not practice proper storage and safety practices.
Teamwork	Notable teamwork shown with a determination to participate/contribute to team success. Completed required individual tasks that contributed to the success of the team.	Teamwork was noted, but was sometimes off task or working on non-related tasks. Contributed to the success of the team, but could have been more engaged to complete tasks sooner.	Notable time off-task with minimal effort given for team success, or did the project alone without relying on others to do their share of the project.	Was not a team player. Either took over project completely, or did not engage in team direction or plans.
Writing/ Reflection	Writing/reflection is very well organized and explained. Student includes all details in design process. Document has almost no grammatical errors.	Writing/reflection is somewhat organized and explained. Student includes most details in design process. Document has very few grammatical errors.	Writing/reflection is not organized and explained. Student includes only a few details in design process. Document has many grammatical errors.	Writing/reflection is incomplete or not turned in. Student includes no details in design process. Document has many grammatical errors.
Presentation	Presentation was well organized and presented in a logical sequence. Presentation reflects a full knowledge of the topic with clear answers and explanations to questions asked.	Presentation was fairly organized and most information presented in a logical sequence. Answers to questions were vague or lacked clarity or accuracy.	Presentation was unorganized and lacked a logical sequence. Presentation reflected little attention to detail. Answers to questions were inaccurate and confusing.	Presentation was not acceptable and reflects a lack of organization or knowledge of the topic. Presentation shows little effort to meet expectations.

Reference

Curriculum Reform Implementation Project. (2006). Impact study 6: Final report. Faculty of Education, Deakin University, Victoria, Australia.

Curriculum Reform Implementation Project. (2003). Outcome-based planning and programming. National Department of Education, Port Moresby, Papua New Guinea: Government Printery.

Curriculum Reform Implementation Project. (2002). Unit 1: Curriculum reform in lower primary curriculum. National Department of Education, Port Moresby, Papua New Guinea: Government Printery.

Czuba. J (2013) Report of the Taskforce for the Review of Outcomes Based Education, Port Moresby PNG

Department of Education (2009), Advance Mathematics Upper Secondary Syllabus, Waigani: DoE

Department of Education (2009), Advance Mathematics Upper Secondary Syllabus, Waigani: DoE

Department of Education (2009), Mathematics Lower Secondary Syllabus, Waigani: DoE

Department of Education, 2003. National Assessment and Reporting Policy. Waigani:DoE.

Jones SB & Couchman K.E., 1981. 3 Unit Mathematics Book 1, Australia, LONGMAN, Pearson Education, Australia. Kaye Stacey, University of Melbourne, Australia

Ley J & Fuller M, 2000. Advance Mathematics Preliminary Course, Australia; Oxford University Press

Prenand G, 1997, Foundation Mathematics, Vol 1, Mathematics Dept, University of Papua New Guinea.

Prenand G, 1997, Foundation Mathematics, Vol 2, Mathematics Dept, University of Papua New Guinea.

Smedley R & Wiseman G, 2001, Introducing Pure Mathematics, 2nd Edition, New York, Oxford University Press

Gateway to the Future_Math 1-3 for Junior High Keirinkan Textbook Co. Japan

Appendices

APPENDIX 1: BLOOM'S TAXONOMY

LEVEL OF UNDERSTANDING	KEY VERBS
CREATING Can the student create a new product or point of view?	Construct, design, and develop, generate, hypothesize, invent, plan, produce, compose, create, make, perform, plan, produce, assemble, formulate,
EVALUATING Can the student justify a stand or decision?	Appraise, argue, assess, choose, conclude, critique, decide, defend, evaluate, judge, justify, predict, prioritize, provoke, rank, rate, select, support, monitor,
ANALYZING Can the student distinguish between the different parts?	Analyzing, characterize, classify, compare, contrast, debate, criticise, deconstruct, deduce, differentiate, discriminate, distinguish, examine, organize, outline, relate, research, separate, experiment, question, test,
APPLYING Can the student use the information in a new way	Apply, change, choose, compute, dramatize, implement, interview, prepare, produce, role play, select, show, transfer, use, demonstrate, illustrate, interpret, operate, sketch, solve, write,
UNDERSTANDING Can the student comprehend ideas or concepts?	Classify, compare, exemplify, conclude, demonstrate, discuss, explain, identify, illustrate, interpret, paraphrase, predict, report, translate, describe, classify,
REMEMBERING Can the student recall or remember the information?	Define, describe, draw, find, identify, label, list, match, name, quote, recall, recite, tell, write, duplicate, memorise, recall, repeat, reproduce, state,

APPENDIX 2: 21ST CENTURY SKILLS

WAYS OF THINKING	Creativity and innovation Think creatively Work creatively with others Implement innovations Critical thinking, problem solving and decision making Reason effectively and evaluate evidence Solve problems Articulate findings Learning to learn and meta-cognition Self-motivation Positive appreciation of learning Adaptability and flexibility
WAYS OF WORKING	Communication Competency in written and oral language Open minded and preparedness to listen Sensitivity to cultural differences Collaboration and teamwork Interact effectively with others Work effectively in diverse teams Prioritise, plan and manage projects
TOOLS FOR WORKING	Information literacy Access and evaluate information Use and manage information Apply technology effectively ICT literacy Open to new ideas, information, tools and ways of thinking Use ICT accurately, creatively, ethically and legally Be aware of cultural and social differences Apply technology appropriately and effectively
LIVING IN THE WORLD	Citizenship – global and local Awareness and understanding of rights and responsibilities as a global citizen Preparedness to participate in community activities Respect the values and privacy of others Personal and social responsibility Communicate constructively in different social situations Understand different viewpoints and perspectives Life and career Adapt to change Manage goals and time Be a self-directed learner Interact effectively with others

APPENDIX 3: TEACHING AND LEARNING STRATEGIES

STRATEGY	TEACHER	STUDENTS
CASE STUDY Used to extend students' understanding of real life issues	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE A method used to increase students' interest, involvement and participation	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points. They are engaged in collaborative learning by delegating and sharing tasks to group members. When debating, they improve their communication skills.
DISCUSSION The purpose of discussion is to educate students about the process of group thinking and collective decision.	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses. Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	Students ponder over the question and answer by providing ideas, experiences and examples. Students participate in the discussion by exchanging ideas with others.
GAMES AND SIMULATIONS Encourages motivation and creates a spirit of competition and challenge to enhance learning	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win
OBSERVATION Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.	Give instructions and monitor every activity students do	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students Observe and ask essential questions Record Interpret

PEER TEACHING & LEARNING (power point presentations, pair learning) Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching.	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class.
PERFORMANCE-RELATED TASKS (dramatization, song/ lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria	Go into their established working groups. Being creative and create dramas, songs/ lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively	Teacher outline the steps and procedures of how to do and the criteria	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

APPENDIX 4: ASSESSMENT STRATEGIES

STRATEGY	DESCRIPTION
ANALOGIES	Students create an analogy between something they are familiar with and the new information they have learned. When asking students to explain the analogy, it will show the depth of their understanding of a topic.
CLASSROOM PRESENTATIONS	A classroom presentation is an assessment strategy that requires students to verbalize their knowledge, select and present samples of finished work, and organize their thoughts about a topic in order to present a summary of their learning. It may provide the basis for assessment upon completion of a student's project or essay.
CONFERENCES	A conference is a formal or informal meeting between the teacher and a student for the purpose of exchanging information or sharing ideas. A conference might be held to explore the student's thinking and suggest next steps; assess the student's level of understanding of a particular concept or procedure; and review, clarify, and extend what the student has already completed
DISCUSSIONS	Having a class discussion on a unit of study provides teachers with valuable information about what the students know about the subject. Focus the discussions on higher level thinking skills and allow students to reflect their learning before the discussion commences.
ESSAYS	An essay is a writing sample in which a student constructs a response to a question, topic, or brief statement, and supplies supporting details or arguments. The essay allows the teacher to assess the student's understanding and/or ability to analyse and synthesize information.
EXHIBITIONS/ DEMONSTRATIONS	An exhibition/demonstration is a performance in a public setting, during which a student explains and applies a process, procedure, etc., in concrete ways to show individual achievement of specific skills and knowledge.
INTERVIEWS	An interview is a face-to-face conversation in which teacher and student use inquiry to share their knowledge and understanding of a topic or problem, and can be used by the teacher to explore the student's thinking; assess the student's level of understanding of a concept or procedure and gather information, obtain clarification, determine positions, and probe for motivations.
LEARNING LOGS	A learning log is an ongoing, visible record kept by a student and recording what he or she is doing or thinking while working on a particular task or assignment. It can be used to assess student progress and growth over time.
OBSERVATION	Observation is a process of systematically viewing and recording students while they work, for the purpose of making programming and instruction decisions. Observation can take place at any time and in any setting. It provides information on students' strengths and weaknesses, learning styles, interests, and attitudes.
PEER ASSESSMENT	Assessment by peers is a powerful way to gather information about students and their understanding. Students can use set criteria to assess the work of their classmates.
PERFORMANCE TASKS	During a performance task, students create, produce, perform, or present works on "real world" issues. The performance task may be used to assess a skill or proficiency, and provides useful information on the process as well as the product.

PORTFOLIOS	A portfolio is a collection of samples of a student's work, and is focused, selective, reflective, and collaborative. It offers a visual demonstration of a student's achievement, capabilities, strengths, weaknesses, knowledge, and specific skills, over time and in a variety of contexts.
QUESTIONS AND ANSWERS (ORAL)	In the question–and-answer strategy, the teacher poses a question and the student answers verbally, rather than in writing. This strategy helps the teacher to determine whether students understand what is being, or has been, presented, and helps students to extend their thinking, generate ideas, or solve problems.
QUIZZES, TESTS, EXAMINATIONS	A quiz, test, or examination requires students to respond to prompts in order to demonstrate their knowledge (orally or in writing) or their skills (e.g., through performance). Quizzes are usually short; examinations are usually longer. Quizzes, tests, or examinations can be adapted for exceptional students and for re-teaching and retesting.
QUESTIONNAIRES	Questionnaires can be used for a variety of purposes. When used as a formative assessment strategy, they provide teachers with information on student learning that they can use to plan further instruction.
RESPONSE JOURNALS	A response journal is a student's personal record containing written, reflective responses to material he or she is reading, viewing, listening to, or discussing. The response journal can be used as an assessment tool in all subject areas.
SELECTED RESPONSES	Strictly speaking a part of quizzes, tests, and examinations, selected responses require students to identify the one correct answer. The strategy can take the form of multiple-choice or true/false formats. Selected response is a commonly used formal procedure for gathering objective evidence about student learning, specifically in memory, recall, and comprehension.
STUDENT SELF-ASSESSMENTS	Self-assessment is a process by which the student gathers information about, and reflects on, his or her own learning. It is the student's own assessment of personal progress in terms of knowledge, skills, processes, or attitudes. Self-assessment leads students to a greater awareness and understanding of themselves as learners.

APPENDIX 5: Standard-based Lesson Plan Template

Strand: ....

Unit: ....

Content Standard: ....

Benchmark: ....

Topic : ....

Lesson Title: ....

Lesson Objective (s): By the end of the lesson, students will be able to;

- ....
- ....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	

Lesson Procedure

Teacher Activity	Student Activity
Introduction	(time in minutes)
Body	(time in minutes)
Modeling	
Guided Practice	
Independent Practice	
Conclusion	(time in minutes)

Assessment/Lesson Evaluation

....

....

APPENDIX 6: Standard based Lesson Plan template - Integrating STEAM

Strand: ....

Unit: ....

Content Standard: ....

Benchmark: ....

Topic : ....

Lesson Title: ....

Lesson Objective (s): By the end of the lesson, students will be able to;

- ....

Essential Questions

- ....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	
STEAM Knowledge and Skills	
Skills	
Knowledge	
STEAM Performance Indicator:	

Lesson Procedure

Teacher Activity	Student Activity
Introduction	(time in minutes)
Body	(time in minutes)
Modeling	
Guided Practice	
Independent Practice	
Conclusion	(time in minutes)

Assessment/Lesson Evaluation

....

APPENDIX 7: Time Allocation

Grades 9 and 10	No. lesson/ wk	Min/week	Grades 11 and 12	No. lessons/ wk	Min/week
English	6	6 x 40 = 240	Applied English	6	6 x 40 = 240
Mathematics	5	5 x 40 = 200	L & L	6	6 x 40 = 240
Science	5	5 x 40 = 200	Advance Math	8	5 x 80 = 400
Social Science	5	5 x 40 = 200	Gen Math	6	8 x 40 = 320
PD	5	5 x 40 = 200	Physics	6	6 x 40 = 240
Business Studies	5	5 x 40 = 200	Biology	6	6 x 40 = 240
Design & Technology	5	5 x 40 = 200	Chemistry	6	6 x 40 = 240
Arts	5	5 x 40 = 200	Applied Science	6	6 x 40 = 240
CCVE	3	3 x 40 = 120	Geology	6	6 x 40 = 240
RI	1	1 x 60 = 60	Geography	6	6 x 40 = 240
Agriculture	5	5 x 40 = 200	History	6	6 x 40 = 240
TOTALS	50	2020min/ wk	Legal Studies	6	6 x 40 = 240
			HPE	6	6 x 40 = 240
			PE	6	6 x 40 = 240
			RE	1	1 x 60 = 60
			Business Studies	6	6 x 40 = 240
			Accounting	6	6 x 40 = 240
			Economics	6	6 x 40 = 240
			Design & Tech	6	6 x 40 = 240
			Computer Studies	6	6 x 40 = 240
			ICT	6	6 x 40 = 240
			CCVE	2	3 x 40 = 120
			ANRM	6	6 x 40 = 240
			TOTALS	128 lessons/ wk	5,460 min/wk

‘FREE ISSUE - NOT FOR SALE’